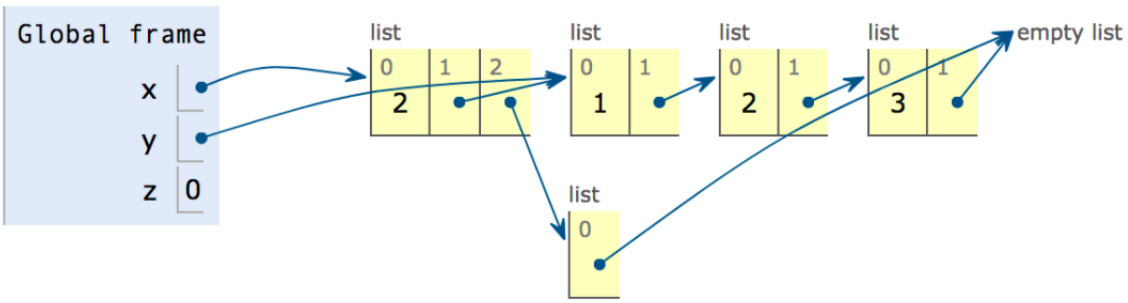
EXAM PREPARATION SECTION 3

LISTS, TREES, AND TREE RECURSION

February 20 to February 22, 2018

1 Lists

1. **Translating a List Diagram to Code**



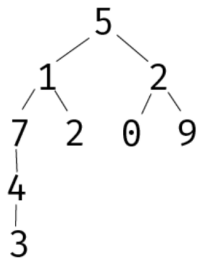
Fill in the following blanks so that after all lines have been executed, the environment looks as in the diagram above. You may not use numerals or mathematical operators in your solution.

```
x, y, z = 1, 2, 3
y = _____________________________________________________________
x = _____________________________________________________________
z = _____________________________________________________________
```

2 Tree Recursion

1. **Tree Recursion with Trees**

   A number `x` is a sum of a tree `t` if and only if there is a path from `t's` root to one of `t's` leaves whose labels sum to `x`. For example, the sums of the tree below are `20 (5+1+7+4+3)`, `8 (5+1+2)`, `7 (5+2+0)`, and `16 (5+2+9)`.



   Fill in the following blanks so that the function behaves as indicated by its docstring. You may assume you have access to the `tree`, `label`, `branches`, and `is_leaf` functions. (See the bottom of Page 4 for the definitions of these `Tree` constructors and selectors.)

```
def sum_range(t):
        """Returns the range of the sums of t, that is, the
           difference between the largest and the smallest
           sums of t."""
        def helper(t):

                if _____________________________________________:

                        return [_______________, _______________]
                else:

                        a = min([_____________________________________])

                        b = max([_____________________________________])

                        x = _____________________________________________

                        return [b + x, a + x]

        x, y = helper(t)

        return x - y
```

2. **This One Goes to Eleven (Fa14 Midterm 2 Q3b)**
Fill in the blanks of the implementation of `no_eleven` below, a function that returns a list of all distinct length-`n` lists of ones and sixes in which 1 and 1 do not appear consecutively.

```
def no_eleven(n):
    """Return a list of lists of 1's and 6's that do not
       contain 1 after 1.

    >>> no_eleven(2)
    [[6, 6], [6, 1], [1, 6]]
    >>> no_eleven(3)
    [[6, 6, 6], [6, 6, 1], [6, 1, 6], [1, 6, 6], [1, 6, 1]]
    >>> no_eleven(4)[:4] # first half
    [[6, 6, 6, 6], [6, 6, 6, 1], [6, 6, 1, 6], [6, 1, 6, 6]]
    >>> no_eleven(4)[4:] # second half
    [[6, 1, 6, 1], [1, 6, 6, 6], [1, 6, 6, 1], [1, 6, 1, 6]]
    """
    if n == 0:

        return _____________________________________________

    elif n == 1:

        return _____________________________________________

    else:

        a, b = no_eleven(_____________), no_eleven(_____________)


        return [__________ for s in a] + [__________ for s in b]
```

3. **Expression Trees (Fa14 Final Q3a)** (Note: This past exam problem has been slightly modified to avoid using Object Oriented Programming.)

   Your partner has created an interpreter for a language that can add or multiply positive integers. Expressions are represented as instances of the `Tree` class and must have one of the following three forms:

   - (**Primitive**) A positive integer `entry` and no branches, representing an integer
   - (**Combination**) The `entry '+'`, representing the sum of the values of its branches
   - (**Combination**) The `entry '*'`, representing the product of the values of its branches

   The sum of no values is 0. The product of no values is 1.

   Unfortunately, multiplication in Python is broken on your computer. Implement `eval_with_add`, which evaluates an expression without using multiplication. You may fill the blanks with names or call expressions, but the only way you are allowed to combine two numbers is using addition.

   You may assume you have access to the `tree`, `label`, `branches`, and `is_leaf` functions, as defined below.

```
# Constructor
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)

# Selectors
def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

# For convenience
def is_leaf(tree):
    return not branches(tree)
```

```
def eval_with_add(t):
    """Evaluate an expression tree of * and + using only
       addition.
    >>> plus = Tree('+', [Tree(2), Tree(3)])
    >>> eval_with_add(plus)
    5
    >>> times = Tree('*', [Tree(2), Tree(3)])
    >>> eval_with_add(times)
    6
    >>> deep = Tree('*', [Tree(2), plus, times])
    >>> eval_with_add(deep)
    60
    >>> eval_with_add(Tree('*'))
    1
    """
    if label(t) == '+':

        return sum(_____________________________________________)

    elif label(t) == '*':


        total = _____________________________________________


        for b in branches(t):


            total, term = 0, _____________________________


            for ___________ in _____________________________:


                total = total + term

        return total

    else:

        return label(t)
```