

## Guerrilla Section 3: Sequences, Data Abstraction, and Trees

### Instructions

Form a group of 3-4. Start on Question 0. Check off with a lab assistant when everyone in your group understands how to solve Question 0. Repeat for Question 1, 2, etc. **You're not allowed to move on from a question until everyone in your group is comfortable with all exercises in the section.** You are allowed to use any and all resources at your disposal, including the interpreter, lecture notes and slides, discussion notes, and labs. You may consult the lab assistants, **but only after you have asked everyone else in your group.** **The purpose of this section is to have all the students working together to learn the material.**

---

### Sequences

#### **Question 0**

Fill out what python would display at each step if applicable.

**Note:** (keep in mind list slicing creates a brand new list, does not modify existing list)

i.

```
>>> lst = [1, 2, 3, 4, 5]
```

```
>>> lst[1:3]
```

```
>>> lst[0:len(lst)]
```

```
>>> lst[-4:]
```

```
>>> lst[:3]
```

```
>>> lst[3:]
```

```
>>> lst[:]
```

ii. **Hint:** You can also specify the increment step-size for slicing. The notation is `lst[start:end:step]`. Remember that a negative step size changes the default start and end.

```
>>> lst[1:4:2]
>>> lst[0:4:3]

>>> lst[:4:2]

>>> lst[1::2]

>>> lst[::2]

>>> lst[::-1]

>>> lst2 = [6, 1, 0, 7]
>>> lst + lst2

>>> lst + 100

>>> lst3 = [[1], [2], [3]]
>>> lst + lst3
```

### Question 1

Draw the environment diagram that results from running the code below

```
def reverse(lst):
    if len(lst) <= 1:
        return lst
    return reverse(lst[1:]) + [lst[0]]
```

```
lst = [1, [2, 3], 4]
rev = reverse(lst)
```

## EXTRA: Question 2

Write `combine_skipper`, which takes in a function `f` and list `lst` and outputs a list. When this function takes in a list `lst`, it looks at the list in chunks of four and applies `f` to the first two elements in every set of four elements and replaces the first element with the output of the function `f` applied to the two elements as the first value and the index of the second item of the original two elements as the second value of the new two elements. `f` takes in a list and outputs a value. [Assume the length of `lst` will always be divisible by four]

```
def combine_skipper(f, lst):
    >>> lst = [4, 7, 3, 2, 1, 8, 5, 6]
    >>> f = lambda l: sum(l)
    >>> lst = combine_skipper(f, lst)
    >>> lst
    [11, 1, 3, 2, 9, 5, 5, 6]
    >>> lst2 = [4, 3, 2, 1]
    >>> lst2 = combine_skipper(f, lst2)
    >>> lst2
    [7, 1, 2, 1]
```

-----

```
while n < len(lst) // 4:
```

-----

```
-----
return lst
```

# STOP!

Don't proceed until everyone in your group has finished and understands all exercises!

## Mutability

### Question 0

- a. Name two data types that are mutable. What does it mean to be mutable?
- b. Name two data types that are not mutable.

### Question 1

- a. Will the following code error? Why?

```
>>> a = 1
>>> b = 2
>>> dt = {a: 1, b: 2}
```

- b. Will the following code error? Why?

```
>>> a = [1]
>>> b = [2]
>>> dt = {a: 1, b: 2}
```

## Question 2

a. Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write "Error", but include all output displayed before the error.

```
>>> a = [1, [2, 3], 4]
>>> c = a[1]
>>> c
```

---

```
>>> a.append(c)
>>> a
```

---

```
>>> c[0] = 0
>>> c
```

---

```
>>> a
```

---

```
>>> a.extend(c)
>>> c[1] = 9
>>> a
```

---

b. Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write "Error", but include all output displayed before the error.

```
>>> lst = [5, 6, 7]
>>> risk = [5, 6, 7]
>>> lst, risk = risk, lst
>>> lst is risk
```

---

```
>>> mist = risk
>>> risk = risk[0:4]
>>> mist.insert(-1, 99)
>>> risk[-1]
```

---

```
# Hint: Try drawing the result of [y + 1 for y in mist] first.
>>> risk = [x for x in [y + 1 for y in mist] if x % 10 != 0]
>>> risk
```

---

```
>>> er = [1, 2]
>>> er.extend(risk.pop())
```

---

# STOP!

Don't proceed until everyone in your group has finished and understands all exercises in this section, and you have gotten checked off!

## Data Abstraction

### **Question 1**

**a.** Why are Abstract Data Types useful?

**b.** What are the two types of functions necessary to make an Abstract Data Type? Describe what they do.

**c.** What is a Data Abstraction Violation?

**d.** Why is it a terrible sin to commit a Data Abstraction Violation?

## Question 2

In lecture, we discussed the rational data type, which represents fractions with the following methods:

- `rational(n, d)` - constructs a rational number with numerator `n`, denominator `d`
- `numer(x)` - returns the numerator of rational number `x`
- `denom(x)` - returns the denominator of rational number `x`

We also presented the following methods that perform operations with rational numbers:

- `add_rationals(x, y)`
- `mul_rationals(x, y)`
- `rationals_are_equal(x, y)`

You'll soon see that we can do a lot with just these simple methods in the exercises below.

a. Write a function that returns the given rational number `x` raised to positive power `e`.

```
from math import pow

def rational_pow(x, e):
    """
    >>> r = rational_pow(rational(2, 3), 2)
    >>> numer(r)
    4
    >>> denom(r)
    9

    >>> r2 = rational_pow(rational(9, 72), 0)
    >>> numer(r2)
    1
    >>> denom(r2)
    1
    """
```

**b.** Implement the following rational number methods.

```
def inverse_rational(x):
    """ Returns the inverse of the given non-zero rational number
    >>> r = rational(2, 3)
    >>> r_inv = inverse_rational(r)
    >>> numer(r_inv)
    3
    >>> denom(r_inv)
    2

    >>> r2 = rational_pow(rational(3, 4), 2)
    >>> r2_inv = inverse_rational(r2)
    >>> numer(r2_inv)
    16
    >>> denom(r2_inv)
    9
    """
```

```

def div_rationals(x, y): # Hint: Use functions defined in Question 2
    """ Returns x / y for given rational x and non-zero rational y
    >>> r1 = rational(2, 3)
    >>> r2 = rational(3, 2)
    >>> div_rationals(r1, r2)
    [4, 9]
    >>> div_rationals(r1, r1)
    [6, 6]
    """

```

c. The irrational number  $e \approx 2.718$  can be generated from an infinite series. Let's try calculating it using our rational number data type! The mathematical formula is as follows:

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

Write a function `approx_e` that returns a rational number approximation of  $e$  to `iter` amount of iterations. We've provided a factorial function.

```

def factorial(n):
    If n == 0:
        return 1
    else:
        return n * factorial(n - 1)

```

```

def approx_e(iter):

```

### Question 3

Assume that `rational`, `numer`, and `denom`, run without error and work like the ADT defined in Question 2. Can you identify where the abstraction barrier is broken? Come up with a scenario where this code runs without error and a scenario where this code would stop working.

---

```
def rational(num, den): # Returns a rational number ADT
    #implementation not shown
def numer(x): # Returns the numerator of the given rational
    #implementation not shown
def denom(x): # Returns the denominator of the given rational
    #implementation not shown
def gcd(a, b): # Returns the GCD of two numbers
    #implementation not shown

def simplify(f1): #Simplifies a rational number
    g = gcd(f1[0], f1[1])
    return rational(numer(f1) // g, denom(f1) // g)

def multiply(f1, f2): # Multiplies and simplifies two rational numbers
    r = rational(numer(f1) * numer(f2), denom(f1) * denom(f2))
    return simplify(r)

x = rational(1, 2)
y = rational(2, 3)
multiply(x, y)
```

---

**STOP!**

Don't proceed until everyone in your group has finished and understands all exercises in this section, and you have gotten checked off!

## Trees

### Question 0

a. Fill in this implementation of a tree:

```
def tree(label, branches = []):
    for b in branches:
        assert is_tree(b), 'branches must be trees'
    return [label] + list(branches)
```

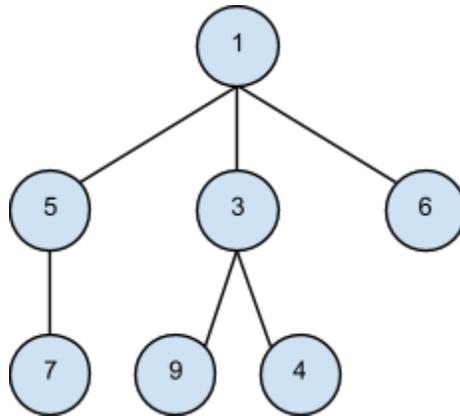
```
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for b in branches(tree):
        if not is_tree(b):
            return False
    return True
```

```
def label(tree):
```

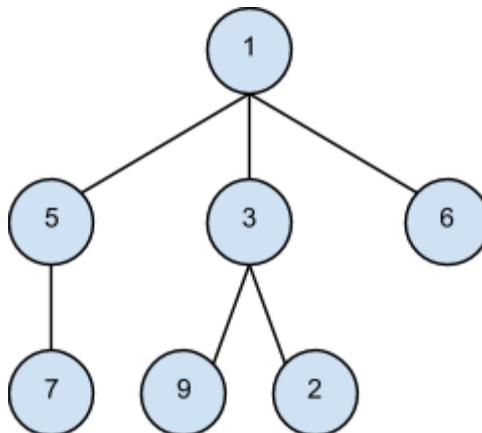
```
def branches(tree):
```

```
def is_leaf(tree):
```

b. A *min-heap* is a tree with the special property that every node's value is less than or equal to the values of all of its children. For example, the following tree is a min-heap:



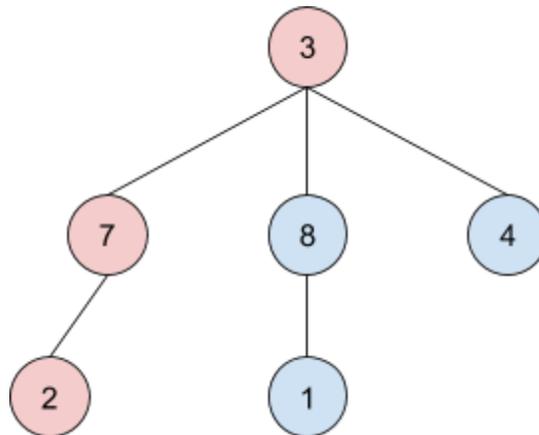
However, the following tree is *not* a min-heap because the node with value 3 has a value greater than one of its children:



Write a function `is_min_heap` that takes a tree and returns `True` if the tree is a min-heap and `False` otherwise.

```
def is_min_heap(t):
```

c. Write a function `largest_product_path` that finds the largest product path possible. A *product path* is defined as the product of all nodes between the root and a leaf. The function takes a tree as its parameter. Assume all nodes have a nonnegative value.



For example, calling `largest_product_path` on the above tree would return 42, since  $3 * 7 * 2$  is the largest product path.

```

def largest_product_path(tree):
    """
    >>> largest_product_path(None)
    0
    >>> largest_product_path(tree(3))
    3
    >>> t = tree(3, [tree(7, [tree(2)]), tree(8, [tree(1)]), tree(4)])
    >>> largest_product_path(t)
    42
    """
    if not _____:
        return 0
    elif is_leaf(tree):
        return _____
    else:
        paths = [ _____ ]
        return _____
  
```

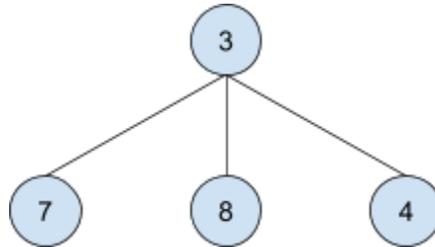
**STOP!**

Don't proceed until everyone in your group has finished and understands all exercises in this section, and you have gotten checked off

## Challenge Question (Optional)

***Come back after finishing everything!***

The *level-order traversal* of a tree is defined as visiting the nodes in each level of a tree before moving onto the nodes in the next level. For example, the level order of the following tree is,



Level-order: 3 7 8 4

a. Write a function `print_level_sorted` that takes in a tree as the parameter and returns a list of the values of the nodes in level order.

```
def level_order(tree):
    """
    >>> t = tree(3, [tree(7, [tree(2, [tree(8), tree(1)]), tree(5)])])
    >>> level_order(t)
    [3 7 5 2 8 1]
    >>> level_order(tree(3))
    [3]
    >>> level_order(None)
    []
    """
    if not _____:
        return []
    current_level, next_level = [label(tree)], [tree]

    while _____:
        find_next = []

        for _____ in _____:
            _____.extend(_____)
        next_level = find_next

        current_level.extend(_____)
    return current_level
```