Guerrilla Section 3: Data Abstraction, Trees, and Growth

Instructions

Form a group of 3-4. Start on Question 0. Check off with a lab assistant when everyone in your group understands how to solve Question 0. Repeat for Question 1, 2, etc. **You're not allowed to move on from a question until you check off with a tutor.** You are allowed to use any and all resources at your disposal, including the interpreter, lecture notes and slides, discussion notes, and labs. You may consult the lab assistants, **but only after you have asked everyone else in your group. The purpose of this section is to have all the students working together to learn the material.**

Sequences

Question 0

Fill out what python would display at each step if applicable.

Note: (keep in mind list slicing creates a brand new list, does not modify existing list)

i.

```
>>> lst = [1, 2, 3, 4, 5]
>>> lst[1:3]
[2, 3]
>>> lst[0:len(lst)]   # one (not so good) way to get the whole tuple
[1, 2, 3, 4, 5]
>>> lst[-4:]         # start: 4th item from end, to the very end
[2, 3, 4, 5]
>>> lst[:3]          # ommiting the left-hand index starts from the
[1, 2, 3]              very beginning
>>> lst[3:]          # ommiting the right-hand index goes to the very
[4, 5]                end
>>> lst[:]           # dupliates the entire tuple
[1, 2, 3, 4, 5]
```

ii. **Hint:** You can also specify the increment step-size for slicing. The notation is lst[start:end:step].

```
>>> lst[1:4:2]     # subsequence from index 1 up to index 4, but only
[2, 4]               getting every other item
>>> lst[0:4:3]     # subsequence from index 0 up to index 4, but only
[1, 4]                getting every third item
>>> lst[:4:2]
[1, 3]
>>> lst[1::2]      # subsequence from index 1 up to end, but only
[2, 4]              getting every other item
>>> lst[::2]       # subsequence from beginning up to end, but only
[1, 3, 5]           getting
every other item

>>> lst[::-1]      # get the entire tuple but in reverse
[5, 4, 3, 2, 1]

>>> lst2 = [6, 1, 0, 7]
>>> lst + lst2
[1, 2, 3, 4, 5, 6, 1, 0, 7]
>>> lst + 100
TypeError: can only concatenate list (not "int") to list
>>> lst3 = [[1], [2], [3]]
>>> lst + lst3
[1, 2, 3, 4, 5, [1], [2] , [3]]
```

Question 1

Draw the environment diagram that results from running the code below
Link: Box and pointer diagram in Python tutor

```
def reverse(lst):
    if len(lst) <= 1:
        return lst
    return reverse(lst[1:]) + [lst[0]]

lst = [1, [2, 3], 4]
rev = reverse(lst)
```

Question 2
Write `combine_skipper`, which takes in a function `f` and list `lst` and outputs a list. When this function takes in a list `lst`, it looks at the list in chunks of four and applies `f` to the first two elements in every set of four elements and replaces the first element with the output of the function `f` applied to the two elements as the first value and the index of the second item of the original two elements as the second value of the new two elements. `f` takes in a list and outputs a value. [Assume the length of `lst` will always be divisible by four]

```
>>> lst = [4, 7, 3, 2, 1, 8, 5, 6]
>>> f = lambda l: sum(l)
>>> lst = combine_skipper(f, lst)
>>> lst
[11, 1, 3, 2, 9, 5, 5, 6]
>>> lst2 = [4, 3, 2, 1]
>>> lst2 = combine_skipper(f, lst2)
>>> lst2
[7, 1, 2, 1]
```

```
def combine_skipper(f, lst):
     n = 0
     while n < len(lst) // 4:
          lst[4*n:4*n+2]=[f(lst[4*n:4*n+2])] + [4*n+1]
          n += 1
     return lst
```

Mutability

Question 0

a. Name two data types that are mutable. What does it mean to be mutable?

Dictionaries, lists

b. Name two data types that are not mutable.

Tuples, functions, int, float

Question 1

a. Will the following code error? If yes, why?

```
>>> a = 1
>>> b = 2
>>> dt = {a: 1, b: 2}
```

No, a and b are immutable types

b. Will the following code error? If yes, why?

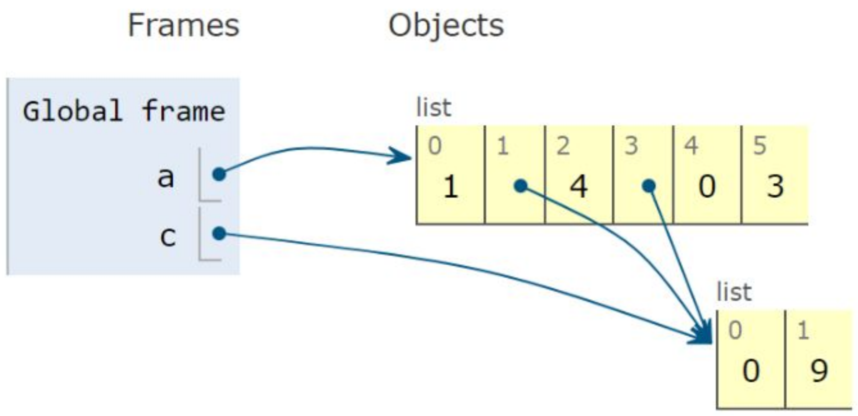
```
>>> a = [1]
>>> b = [2]
>>> dt = {a: 1, b: 2}
```

Yes, a and b are mutable types

Question 1

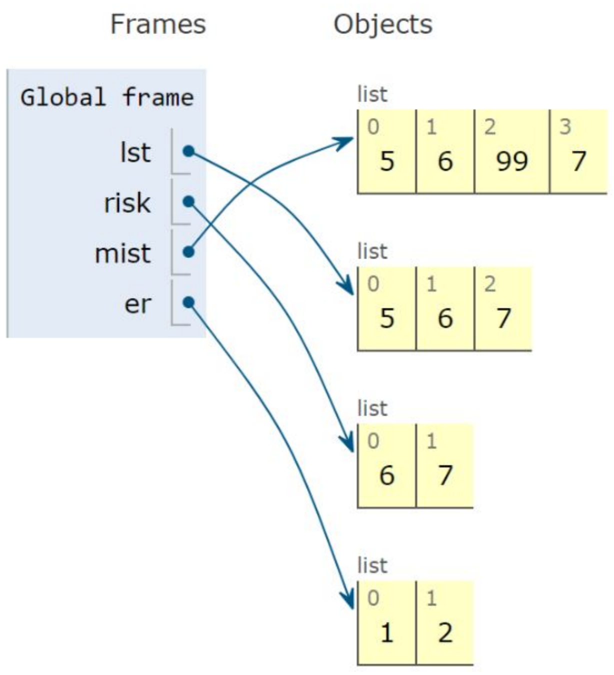
a. Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write “Error”, but include all output displayed before the error.

```
>>> a = [1, [2, 3], 4]
>>> c = a[1]
>>> c
[2, 3]
>>> a.append(c)
>>> a
[1, [2, 3], 4, [2, 3]]
>>> c[0] = 0
>>> c
[0, 3]
>>> a
[1, [0, 3], 4, [0, 3]]
>>> a.extend(c)
>>> c[1] = 9
>>> a
[1, [0, 9], 4, [0, 9], 0, 3]
```



b. Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write “Error”, but include all output displayed before the error.

```
>>> lst = [5, 6, 7]
>>> risk = [5, 6, 7]
>>> lst, risk = risk, lst
>>> lst is risk
False
>>> mist = risk
>>> risk = risk[0:4]
>>> mist.insert(-1, 99)
>>> risk[-1]
7
# Hint: Try drawing the result of [y + 1 for y in mist] first.
>>> risk = [x for  x in [y + 1 for y in mist] if x % 10 != 0]
>>> risk
[6, 7, 8]
>>> er = [1, 2]
>>> er.extend(risk.pop())
Error
```



Data Abstraction

Question 1

a. Why are Abstract Data Types useful?

- More readable code.
    - Constructors and selectors have human-readable names.
- Makes collaboration easier.
    - Other programmers don't have to worry about implementation details.
- Prevents error propagation.
    - Fix errors in a single function rather than all over your program.

b. What are the two types of functions necessary to make an Abstract Data Type? Describe what they do.

- Constructors make the ADT.
- Selectors take instances of the ADT and output relevant information stored in it.

c. What is a Data Abstraction Violation?

Put simply, a Data Abstraction Violation is when you bypass the constructors and selectors for an ADT, and directly use how it's implemented in the rest of your code, thus assuming that its implementation will not change.

d. Why is it a terrible sin to commit a Data Abstraction Violation?

We cannot assume we know how the ADT is constructed except by using constructors and likewise, we cannot assume we know how to access details of our ADT except through selectors. The details are supposed to be abstracted away by the constructors and selectors. If we bypass the constructors and selectors and access the details directly, any small change to the implementation of our ADT could break our entire program.

Question 2

In lecture, we discussed the rational data type, which represents fractions with the following methods:
• `rational(n, d)` - constructs a rational number with numerator n, denominator d •
`numer(x)` - returns the numerator of rational number x
• `denom(x)` - returns the denominator of rational number x

We also presented the following methods that perform operations with rational numbers:

- `add_rationals(x, y)`
- `mul_rationals(x, y)`
- `rationals_are_equal(x, y)`

You'll soon see that we can do a lot with just these simple methods in the exercises below.

a. Write a function that returns the given rational number x raised to positive power e.

```
from math import pow

def rational_pow(x, e):
    """
    >>> r = rational(2, 3)
    >>> rational_pow(r, 2)
    [4, 9]
    >>> rational_pow(r, 0)
    [1, 1]
    """
    return rational(pow(numer(x), e), pow(denom(x), e))
```

b. Implement the following rational number methods.

```
def inverse_rational(x):
    """ Returns the inverse of the given non-zero rational number
    >>> r = rational(2, 3)
    >>> inverse_rational(r)
    [9, 4]
    >>> rational_pow(rational(10, 9))
    [9, 10]
    """
    return rational(denom(x), numer(x))
```

```
def div_rationals(x, y): # Hint: Use functions defined in Question 2
    """ Returns x / y for given rational x and non-zero rational y
    >>> r1 = rational(2, 3)
    >>> r2 = rational(3, 2)
    >>> div_rationals(r1, r2)
    [4, 9]
    >>> div_rationals(r1, r1)
    [6, 6]
    """
    return mul_rationals(x, inverse_rational(y))
```

c. The irrational number e ≈ 2.718 can be generated from an infinite series. Let's try calculating it using our rational number data type! The mathematical formula is as follows:

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

Write a function approx_e that returns a rational number approximation of e to iter amount of iterations. We've provided a factorial function.

```
def factorial(n):
    If n == 0:
        return 1
    else:
        return n * factorial(n - 1)
```

```
def approx_e(iter):
    k = 0
    e = rational(0, 1)
    while k < iter:
        e = add_rationals(e, rational(1, factorial(k)))
        k += 1
    return e
```

Question 3

Assume that `rational,` `numer,` and `denom`, run without error and work like the ADT defined in Question 2. Can you identify where the abstraction barrier is broken? Come up with a scenario where this code runs without error and a scenario where this code would stop working.

```
def rational(num, den): # Returns a rational number ADT
    #implementation not shown
def numer(x): # Returns the numerator of the given rational
    #implementation not shown
def denom(x): # Returns the denominator of the given rational
    #implementation not shown
def gcd(a, b): # Returns the GCD of two numbers
    #implementation not shown

def simplify(f1): #Simplifies a rational number
    g = gcd(f1[0], f1[1])
    return rational(numer(f1) // g, denom(f1) // g)

def multiply(f1, f2): # Multiples and simplifies two rational numbers
    r = rational(numer(f1) * numer(f2), denom(f1) * denom(f2))
    return simplify(r)

x = rational(1, 2)
y = rational(2, 3)
multiply(x, y)
```

The abstraction barrier is broken inside simplify(f1) when calling gcd(f1[0], f1[1]). This assumes rational returns a type that can be indexed through. *i.e. This would work if rational returned a list. However, this would not work if rational returned a dictionary.*

The correct implementation of simplify would be

```
def simplify(f1):
    g = gcd(numer(x), denom(x))
    return rational(numer(f1) // g, denom(f1) // g)
```

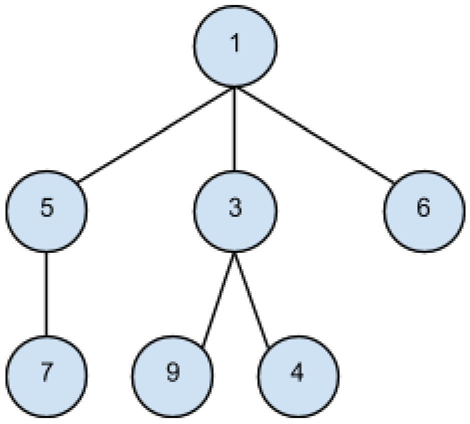
Trees

Question 0

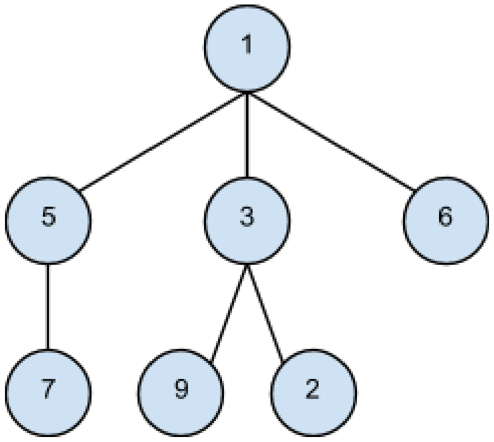
a. Fill in this implementation of a tree:

```
def tree(label, branches = []):
    for b in branches:
        assert is_tree(b), 'branches must be trees'
    return [label] + list(branches)



def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for b in branches(tree):
        if not is_tree(b):
            return False
    return True




def label(tree):
    return tree[0]



def branches(tree):
    return tree[1:]



def is_leaf(tree):
    return not branches(tree)
```

b. A *min-heap* is a tree with the special property that every node's value is less than or equal to the values of all of its children. For example, the following tree is a min-heap:



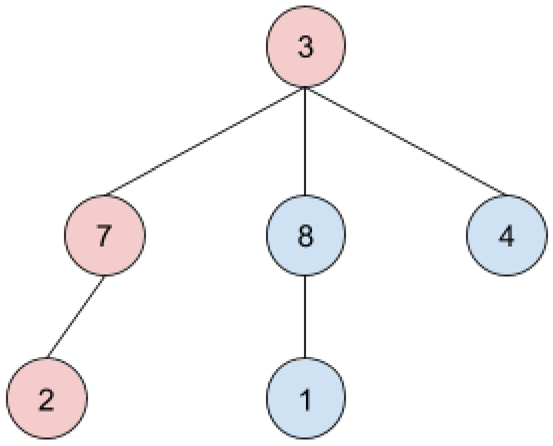
However, the following tree is *not* a min-heap because the node with value 3 has a value greater than one of its children:



Write a function `is_min_heap` that takes a tree and returns `True` if the tree is a min-heap and `False` otherwise.

```
def is_min_heap(t):
    for b in branches(t):
        if label(t) > label(b) or not is_min_heap(b):
            return False
    return True
```

c. Write a function `largest_product_path` that finds the largest product path possible. A *product path* is defined as the product of all nodes between the root and a leaf. The function takes a tree as its parameter. Assume all nodes have a nonnegative value.



For example, calling `largest_product_path` on the above tree would return 42, since 3 * 7 * 2 is the largest product path.

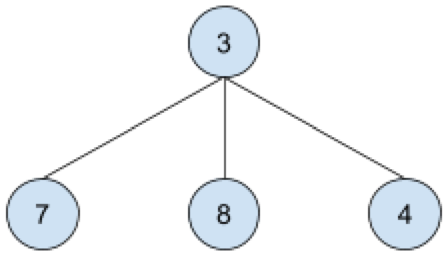
```
def largest_product_path(tree):
    """
    >>> largest_product_path(None)
    0
    >>> largest_product_path(tree(3))
    3
    >>> t = tree(3, [tree(7, [tree(2)]), tree(8, [tree(1)]), tree(4)])
    >>> largest_product_path(t)
    42
    """
    if not tree:
        return 0
    elif is_leaf(tree):
        return label(tree)
    else:
        paths = [largest_product_path(t) for t in branches(tree)]
        return label(tree) * max(paths)
```

STOP!

Don't proceed until everyone in your group has finished and understands all exercises in this section, and you have gotten checked off

(Optional) Challenge Question
Come back after finishing everything!

The *level-order traversal* of a tree is defined as visiting the nodes in each level of a tree before moving onto the nodes in the next level. For example, the level order of the following tree is,



Level-order: 3 7 8 4

a. Write a function print_level_sorted that takes in a tree as the parameter and returns a list of the values of the nodes in level order.

```
# Iterative implementation
def level_order(tree):
    if not tree:
        return []
    current_level, next_level = [label(tree)], [tree]
    while next_level:
        find_next= []
        for b in next_level:
            find_next.extend(branches(b))
        next_level = find_next
        current_level.extend([label(t) for t in next_level])
    return current_level


# Recursive implementation
def level_order(tree):
    def find_next(current_level):
        if current_level == []:
            return []
        else:
            next_level = []
            for b in current_level:
                next_level.extend(branches(b))
            return [label(t) for t in next_level] + find_next(next_level)
    return [label(tree)] + find_next([tree])
```