TREES AND ORDERS OF GROWTH

COMPUTER SCIENCE MENTORS 61A

October 2 to October 6, 2017

1 Trees

Things to remember:

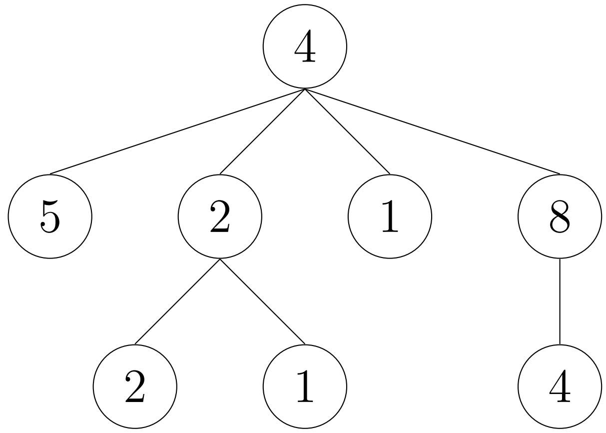
```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```

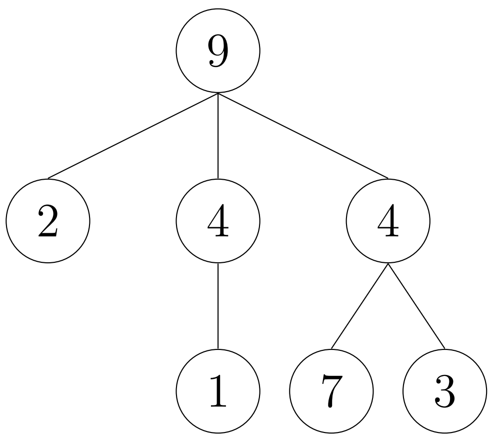
1. Draw the tree that is created by the following statement:

```
tree(4,
     [tree(5, []),
      tree(2,
          [tree(2, []),
           tree(1, [])]),
      tree(1, []),
      tree(8,
          [tree(4, [])])])
```



Solution:

2. Construct the following tree and save it to the variable `t`.



Solution:

```
t = tree(9, [tree(2, []),
             tree(4, [tree(1, [])]),
             tree(4, [tree(7, []),
                      tree(3, [])])])
```

3. What would this output?

```
>>> label(t)
```

Solution: 9

```
>>> branches(t)[2]
```

Solution:

```
tree(4, [tree(7, []), tree(3, [])])
```

```
>>> branches(branches(t)[2])[0]
```

Solution:

```
tree(7, [])
```

4. Write the Python expression to return the integer 2 from t.

Solution:

```
label(branches(t)[0])
```

5. Write the function `sum_of_nodes` which takes in a tree and outputs the sum of all the elements in the tree.

```
def sum_of_nodes(t):
    """
    >>> t = tree(...) # Tree from question 2.
    >>> sum_of_nodes(t) # 9 + 2 + 4 + 4 + 1 + 7 + 3 = 30
    30
    """
```

Solution:

```
    total = label(t)
    for branch in branches(t):
        total += sum_of_nodes(branch)
    return total

    Alternative solution:
    return label(t) +\
           sum([sum_of_nodes(b) for b in branches(t)])
```

2 Orders of Growth

6. In big-Θ notation, what is the runtime for `foo`?

   (a)
   ```
   def foo(n):
       for i in range(n):
           print('hello')
   ```

   Solution: $O(n)$. This is simple loop that will run n times.

   (b) What's the runtime of `foo` if we change `range(n)`:

   i. To `range(n / 2)`?

   Solution: $O(n)$. The loop runs $n/2$ times, but we ignore constant factors.

   ii. To `range(10)`?

   Solution: $O(1)$. No matter the size of n, we will run the loop the same number of times.

   iii. To `range(10000000)`?

   Solution: $O(1)$. No matter the size of n, we will run the loop the same number of times.

7. What is the order of growth in time for the following functions? Use big-Θ notation.

   (a)
   ```
   def strange_add(n):
       if n == 0:
           return 1
       else:
           return strange_add(n - 1) + strange_add(n - 1)
   ```

   Solution: $\Theta(2^n)$. To see this, try drawing out the call tree. Each level will create two new calls to `strange_add`, and there are n levels. Therefore, 2^n calls.

   (b)
   ```
   def stranger_add(n):
   ```

Computer Science Mentors CS61A Fall 2017: Paul Bitutsky and Jason Goodman, with
Colin Schoen, Kevin Lin, Thomas Zhang, Danelle Nachum, Jennie Chen, Nipun Ramakrishnan, Christopher Allsman, Nikunj Jain, Shreya Sahoo, Joshua Zeitsoff

```
    if n < 3:
        return n
    elif n % 3 ==  0:
        return stranger_add(n - 1) + stranger_add(n - 2) +
           stranger_add(n - 3)
    else:
        return n
```

Solution: $\Theta(n)$ is n is a multiple of 3, otherwise $\Theta(1)$.
The case where n is not a multiple of 3 is fairly obvious – we step into the else clause and immediately return.
If n is a multiple of 3, then neither $n-1$ nor $n-2$ are multiples of 3 so those calls will take constant time. Therefore, we just run `stranger_add`, decrementing the argument by 3 each time.

(c)
```
def waffle(n):
    i = 0
    total = 0
    while i < n:
        for j in range(50 * n):
            total += 1
        i += 1
    return total
```

Solution: $\Theta(n^2)$. Ignore the constant term in $50 * n$, and it because just two for loops.

(d)
```
def belgian_waffle(n):
    i = 0
    total = 0
    while i < n:
        for j in range(n ** 2):
            total += 1
        i += 1
    return total
```

Solution: $\Theta(n^3)$. Inner loop runs n^2 times, and the outer loop runs n times. To get the total, multiply those together.

(e)
```
def pancake(n):
    if n == 0 or n == 1:
        return n
    # Flip will always perform three operations and return
       -n.
    return flip(n) + pancake(n - 1) + pancake(n - 2)
```

Solution: $\Theta(2^n)$. Flip will run in constant time. Therefore, this call tree looks very similar to fib! (which is 2^n)

(f)
```
def toast(n):
    i = 0
    j = 0
    stack = 0
    while i < n:
        stack += pancake(n)
        i += 1
```

```
        while j < n:
            stack += 1
            j += 1
        return stack
```

Solution: $\Theta(n2^n)$. There are two loops: the first runs n times for 2^n calls each time (due to pancake), for a total of $n2^n$. The second loop runs n times. When calculating orders of growth however, we focus on the dominating term – in this case, $n2^n$.

8. Consider the following functions:

```
def hailstone(n):
    print(n)
    if n < 2:
        return
    if n % 2 == 0:
        hailstone(n // 2)
    else:
        hailstone((n * 3) + 1)

def fib(n):
   if n < 2:
      return n
   return fib(n - 1) + fib(n - 2)

def foo(n, f):
    return n + f(500)
```

In big-Θ notation, describe the runtime for the following:

(a) `foo(10, hailstone)`

Solution: $\Theta(1)$. $f(500)$ is independent of the size of the input n.

(b) `foo(3000, fib)`

Solution: $\Theta(1)$. See above.

9. **Orders of Growth and Trees:** Assume we are using the non-mutable tree implementation introduced in discussion. Consider the following function:

```
def word_finder(t, p, word):
    if root(t) == word:
        p -= 1
        if p == 0:
            return True
    for branch in branches(t):
        if word_finder(branch, p, word):
            return True
    return False
```

(a) What does this function do?

> **Solution:** This function take a Tree `t`, an integer `n`, and a string `word` in as input.
>
> Then, `word_finder` returns True if any paths from the root towards the leaves have at least `n` occurrences of the word and False otherwise.

(b) If a tree has n total nodes, what is the total runtime in big-Θ notation?

> **Solution:** $\Theta(n)$. At worst, we must visit every node of the tree.