Announcements
Names, Assignment, and User-Defined Functions

(Demo)
Types of Expressions
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Primitive expressions:
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Primitive expressions:

2

Number or Numeral
Types of Expressions

**Primitive expressions:**

- Number or Numeral
- Name

2
add
Types of Expressions

Primitive expressions:

- 2
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  - Name
- 'hello'
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Types of Expressions

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**Call expressions:**
Types of Expressions

**Primitive expressions:**
- 2
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**Call expressions:**
- `max ( 2 , 3 )`
Types of Expressions

**Primitive expressions:**
- 2  
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**Call expressions:**
- `max` ( 2 , 3 )  
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Types of Expressions

**Primitive expressions:**

- 2
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- **Operator**
- **Operand**
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**Call expressions:**

- max
  - **Operator**
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Types of Expressions

Primitive expressions:
- 2
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Call expressions:
- max(2, 3)
- max(min(pow(3, 5), -4), min(1, -2))
Types of Expressions

Primitive expressions:

<table>
<thead>
<tr>
<th>Operator</th>
<th>Operand 1</th>
<th>Operand 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>add</td>
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Call expressions:

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max(2, 3)
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An operand can also be a call expression

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max(min(pow(3, 5), -4), min(1, -2))
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Types of Expressions

### Primitive expressions:

- **Number or Numeral**: 2
- **Name**: `add`
- **String**: 'hello'

An operand can also be a call expression.

### Call expressions:

- **Operator**: `max`
- **Operand**: (2, 3)
- **Operand**: `max(min(pow(3, 5), -4), min(1, -2))`

An operand can also be a call expression.
Discussion Question 1
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What is the value of the final expression in this sequence?
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>>> f = min
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>>> g, h = min, max
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5
Discussion Question 1

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Discussion Question 1

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Environment diagrams visualize the interpreter’s process.
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2. tau = 2 * pi
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Code (left): Frames (right):

Global frame

pi 3.1416
Environment Diagrams

Environment diagrams visualize the interpreter’s process.

Statement 1: `from math import pi`
Statement 2: `tau = 2 * pi`

Code (left): Statements and expressions

Frames (right):
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- pi: 3.1416
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Interactive Diagram
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| pi | 3.1416 |

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Interactive Diagram
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Arrows indicate evaluation order

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<td></td>
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**Code (left):**

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(Demo)

Interactive Diagram
Assignment Statements

Interactive Diagram
Assignment Statements

1  a = 1
2  b = 2
3  b, a = a + b, b
Assignment Statements

1. \(a = 1\)
2. \(b = 2\)
3. \(b, a = a + b, b\)

Interactive Diagram

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>a</td>
</tr>
<tr>
<td>b</td>
</tr>
</tbody>
</table>
Assignment Statements

1. `a = 1`
2. `b = 2`
3. `b, a = a + b, b`

Interactive Diagram

Global frame

<p>| | |</p>
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1. $a = 1$

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Interactive Diagram

Global frame

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Execution rule for assignment statements:
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1. Evaluate all expressions to the right of = from left to right.
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1. Evaluate all expressions to the right of = from left to right.

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Discussion Question 1 Solution

(Demo)

Interactive Diagram
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1 \( f = \text{min} \)
2 \( f = \text{max} \)
3 \( g, h = \text{min}, \text{max} \)
4 \( \text{max} = g \)
5 \( \text{max}(f(2, g(h(1, 5), 3)), 4) \)
Discussion Question 1 Solution

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2 \( f = \max \)
3 \( g, h = \min, \max \)
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(Demo)

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(Demo)

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(Demo)

Interactive Diagram
Discussion Question 1 Solution

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(Demo)

Interactive Diagram
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(Demo)

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(Demo)

Global frame

\[
\begin{align*}
\text{func max(...)} & \quad f \\
\text{func min(...)} & \quad h \\
\text{func max(...)} & \quad g \\
\text{func min(...)} & \quad \text{max}
\end{align*}
\]

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Interactive Diagram
Defining Functions
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Assignment is a simple means of abstraction: binds names to values

Function definition is a more powerful means of abstraction: binds names to expressions
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```python
>>> def <name>(<formal parameters>):
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Function *body* defines the computation performed when the function is applied.
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1. Create a function with signature \texttt{<name>\(<formal parameters>\)}
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Calling User-Defined Functions

Interactive Diagram
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Procedure for calling/applying user-defined functions (version 1):

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1 from operator import mul
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Interactive Diagram
Looking Up Names In Environments

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So far, the current environment is either:
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