Lecture #2: Functions, Expressions, Environments
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From Last Time

• From last lecture: *Values* are data we want to manipulate and in particular,

• *Functions* are values that perform computations on values.

• *Expressions* denote computations that produce values.

• Today, we’ll look at them in some detail at how functions operate on data values and how expressions denote these operations.

• As usual, although our concrete examples all involve Python, the actual concepts apply almost universally to programming languages.
Functions

• For this lecture, we’re going to use this notation to show function values (which are created by evaluating function definitions):

  \[
  \text{abs(number):} \quad \text{add(left, right)}
  \]

  (We’ll simplify this in a bit to make it easier to write.)

• The green parenthesized lists indicate the number of parameter values or inputs the functions operate on (this information is also known as a function’s signature).

• For our purposes, the blue name is simply a helpful comment to suggest what the function does, and the specific (green) parameter names are likewise just helpful hints.

• (Python actually maintains this intrinsic name and the parameter names internally, but this is not a universal feature of programming languages.)
Pure Functions

• The fundamental operation on function values is to call or invoke them, which means giving them one value for each formal parameter and having them produce the result of their computation on these values:

-5 ▷ \text{abs(number):} \quad \text{▷ 5}

(29, 13) ▷ \text{add(left, right)} \quad \text{▷ 42}

• These two functions are pure: their output depends only on their input parameters’ values, and they do nothing in response to a call but compute a value.
Impure Functions

- Functions may do additional things when called besides returning a value.
- We call such things *side effects*.
- Example: the built-in `print` function:

  \[
  -5 \rightarrow \text{print}(\bullet \bullet \bullet) \rightarrow \text{None}
  \]

  *display text '-5'*

- Displaying text is `print`'s side effect. Its value, in fact, is generally useless (always the null value).
Other Kinds of Impurity

• Most side-effects involve changing the value of some variable.

• Example: the function `random.randint`:

```python
>>> random.randint(0, 100)  # Random number in 0--100.
13
>>> random.randint(0, 100)
55  # Something must have changed!
```
Call Expressions

• A call expression denotes the operation of calling a function.

• Consider \texttt{add}(2, 3):

\[
\text{add}(2, 3)
\]

\[
\begin{array}{c}
\text{Operator} \\
\text{Operand 0} \\
\text{Operand 1}
\end{array}
\]

• The operator and the operands are all themselves expressions (recursion again).

• To evaluate this call expression:
  - Evaluate the operator (let’s call the value \( C \));
  - Evaluate the operands in the order they appear (let’s call the values \( P_0 \) and \( P_1 \))
  - Call \( C \) (which must be a function) with parameters \( P_0 \) and \( P_1 \).

• Together with the definitions for base cases (mostly literal expressions and symbolic names), this describes how to evaluate any call.
Example: From Expression to Value

Let's evaluate the expression `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`. In the following sequence, values are shown in boxes. Everything outside a box is an expression.

- `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`
- `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`
- `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`
- `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`
- `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`
\[ \text{mul}(\text{left, right}) \]

- \( (26, \text{add}(0x3, 005)) \)

- \( (26, (3, 5)) \)

- \( (26, 8) \)

- 208.
Example: Print

What about an expression with side effects?

1. print(print(1), print(2))
2. ( (1), print(2))
3. (None, print(2))
   \hspace{1cm} \text{and print '1'.}
4. (None, (2))
5. (None, None)
   \hspace{1cm} \text{and print '2'.}
6. None
   \hspace{1cm} \text{and print 'None None'.}
Names

• Evaluating expressions that are literals is easy: the literal’s text gives all the information needed.
• But how did I evaluate names like add, mul, or print?
• Deduction: there must be another source of information.
• We’ll first try a simple approach: substitution of values for names.
• This won’t cover all the cases, however, and so we’ll introduce the concept of an environment.
Substitution

• Let’s try to explain the effect of

\[
\begin{align*}
  x &= 3 \\
  y &= x \times 2 \\
  z &= y \times x
\end{align*}
\]

by treating each assignment (=) as a **definition**.

• Thus, we get

\[
\begin{align*}
  x &= 3 & x &= 3 & x &= 3 & x &= 3 \\
  y &= x \times 2 & y &= 3 \times 2 & y &= 6 & y &= 6 \\
  z &= y \times x & z &= y \times 3 & z &= 6 \times 3 & z &= 216
\end{align*}
\]

• That is, we **replace names by their definitions (values)**.
Substitution and Functions

• Now consider a simple function definition:

```python
def compute(x, y):
    return (x * y) ** x

print(compute(3, 2))
```

• A `def` statement is sort of like an assignment, but specialized to functional values.

• The `def` statement above defines `compute` to be “the function of `x` and `y` that returns \((xy)^x\).”

• Here, I’ll use a common notation for that (due to Church):

\[
\lambda x, y : (xy)^x.
\]

• So after substitution for `compute`, we have

\[
\text{print}((\lambda x, y : (xy)^z)(3, 2))
\]

• Now what?
Substitution and Formal Parameters

• A function call such as
  \[(\lambda \ x, \ y : \ (xy)^2) \ (3, \ 2)\]
  from last slide is like a *simultaneous assignment* to or substitution for \(x\) and \(y\).

• So we replace the whole expression with
  \[(3 \cdot 2)^3\]
  and (eventually), just 216.
Getting Fancy

- What about this?
  ```python
def incr(n):
    def f(x):
        return n + x
    return f
  
  print(incr(5)(6))
```

- The `incr` function returns a function. The argument to `print` then calls this function on 6.

- What happens?
Answer

- First, deal with `incr`:

  ```python
  def incr(n):
      def f(x):
          return n + x
      return f

  print/incr(5)(6))
  print((\lambda n: \ return \ \lambda x: n + x)(5)(6))
  ```

- The 5 now gets substituted for `n`:

  ```python
  print((\lambda x: 5 + x)(6)
  ```

- And 6 for `x`:

  ```python
  print(5 + 6)
  ```

- Finally giving

  ```python
  print(11)
  ```
Trouble

- Alas, this relatively simple (if tedious) approach doesn’t work.
- Example:

  ```
  x = 4
  x = 8
  print(x)
  ```

- If we just substitute for the first `x` as before:

  ```
  x = 4
  x = 8  # or even worse: 4 = 8
  print(4)
  ```

- …we get a wrong result (4 instead of 8).
- After one substitution, `x` isn’t around any more to substitute for.
- We need something stronger.
Environments

• An environment is a mapping from names to values.
• We say that a name is bound to a value in this environment.
• In its simplest form, it consists of a single global environment frame:

```
from math import pi
radius = 10
def square(x): return x**2
```

Pre-defined

- abs
- pi: 3.1415926
- square

Imported

- from math import pi

Assigned

- radius: 10

Assigned by def

- def square(x): return x**2

Global

- abs:
- pi: 3.1415926
- radius: 10
- square:

abs(x)

square(x)

return mul(x, x)
You’ll be using the Python Tutor from time to time, which uses a somewhat different notation for function values. Might as well get used to it (we’ll explain the “parent=” stuff in a later lecture):

```python
from math import pi
radius = 10
def square(x): return x**2
```

**Global**
- `abs`: ...
- `pi`: 3.1415926 ...
- `radius`: 10 ...
- `square`: ...

**Pre-defined**
- `func abs(x) [parent=Global]`

**Imported**
- `func pi: 3.1415926 [parent=Global]`

**Assigned**
- `func radius: 10 [parent=Global]`

**Assigned by def**
- `func square(x) [parent=Global]`
Environments and Evaluation

• Every expression is evaluated in an environment, which supplies the meanings of any names in it.

• Evaluating an expression typically involves first evaluating its subexpressions (the operators and operands of calls, the operands of conventional expressions such as $x*(y+z)$, ...).

• These subexpressions are evaluated in the same environment as the expression that contains them.

• Once their subexpressions (operator + operands) are evaluated, calls to user-defined functions must evaluate the expressions and statements from the definition of those functions.
Evaluating User-Defined Function Calls

- Consider the expression `square(mul(x, x))` after executing

```python
from operator import mul
def square(x):
    return mul(x, x)
x = -2
```

![Diagram showing the evaluation process of `square(mul(x, x))` with global variables and function definitions.]
Evaluating User-Defined Function Calls (II)

- First evaluate the subexpressions of $\text{square}(\text{mul}(x, x))$ in the global environment:

- Evaluating subexpressions $x$, $\text{mul}$, and $\text{square}$ take values from the expression's environment.
Evaluating User-Defined Functions Calls (III)

• Then perform the primitive multiply function:

```
Global
mul:
  ...
  x:
  ...
square:

  func mul(L,R[parent=Global])
  func square(x)[parent=Global]
  square(4)
```
Evaluating User-Defined Functions Calls (IV)

- To explain parameter to user-defined square function, extend environment with a local environment frame, attached to the frame in which square was defined (the global one in this case), and giving \( x \) the operand value.

- Now replace original call with evaluating body of square in the new local environment.
Evaluating User-Defined Functions Calls (V)

- When we evaluate `mul(x, x)` in this new environment, we get the same value as before for `mul`, but the local value for `x`.
- When evaluating an identifier in a chain of environments, follow the parent environment links to the first frame containing its definition.
So How Does This Help?

• The original problem that led to this whole environment diagram thing was how to deal with:

```python
x = 4
x = 8
print(x)
```

• Now it's easy. Each time we assign to `x`, we create a new binding for it in the current evaluation frame (replacing the old one, if any).

• We get the new (last assigned) value when we look up `x` in the modified environment.