Announcements
Recursive Functions
Recursive Functions

Demo
Recursive Functions

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**Implication:** Executing the body of a recursive function may require applying that function.
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Digit Sums

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Digit Sums

2 + 0 + 1 + 6 = 9

- If a number \( a \) is divisible by 9, then \( \text{sum\_digits}(a) \) is also divisible by 9
- Useful for typo detection!
Digit Sums

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The Bank of 61A
1234 5678 9098 7658
OSKI THE BEAR
Digit Sums

If a number $a$ is divisible by 9, then $\text{sum_digits}(a)$ is also divisible by 9

Useful for typo detection!

\[2 + 0 + 1 + 6 = 9\]

A checksum digit is a function of all the other digits; it can be computed to detect typos.
Digit Sums

If a number $a$ is divisible by 9, then $\text{sum_digits}(a)$ is also divisible by 9

Useful for typo detection!

$2 + 0 + 1 + 6 = 9$

Credit cards actually use the Luhn algorithm, which we'll implement after $\text{sum_digits}$
The Problem Within the Problem

The sum of the digits of 6 is 6.

Likewise for any one-digit (non-negative) number (i.e., \(< 10\)).

The sum of the digits of 2016 is

\[
\text{Sum of these digits} + \text{This digit}
\]

That is, we can break the problem of summing the digits of 2016 into a smaller instance of the same problem, plus some extra stuff.

We call this recursion
Sum Digits Without a While Statement
def split(n):
    """Split positive n into all but its last digit and its last digit."""
    return n // 10, n % 10
Sum Digits Without a While Statement

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    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

```bash
  $ python
  >>> sum_digits(123)
  6
  >>> sum_digits(1020)
  3
  >>> sum_digits(0)
  0
```

Sum Digits Without a While Statement

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The Anatomy of a Recursive Function

The def statement header is similar to other functions

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• Conditional statements check for base cases

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- The **def statement header** is similar to other functions
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- Base cases are evaluated without recursive calls

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The Anatomy of a Recursive Function

- The \texttt{def} statement header is similar to other functions
- Conditional statements check for \texttt{base cases}
- Base cases are evaluated \texttt{without} recursive calls

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def sum_digits(n):
    
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    if \texttt{n < 10}:
        \texttt{return n}

    else:
        all_but_last, last = split(n)

        \texttt{return sum_digits(all,but, last) + last}
```
The Anatomy of a Recursive Function

• The `def` statement header is similar to other functions
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(Demo)
Recursion in Environment Diagrams
Recursion in Environment Diagrams

1    def fact(n):
2        if n == 0:
3            return 1
4        else:
5            return n * fact(n-1)
6
7    fact(3)
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```

- The same function `fact` is called multiple times

(Demo)

Global frame

- `fact`

f1: `fact [parent=Global]`

```
    n  3
```

f2: `fact [parent=Global]`

```
    n  2
```

f3: `fact [parent=Global]`

```
    n  1
```

f4: `fact [parent=Global]`

```
    n  0
    Return value  1
```
Recursion in Environment Diagrams

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(Demo)

```
Global frame

func fact(n) [parent=Global]

f1: fact [parent=Global]
    n 3

f2: fact [parent=Global]
    n 2

f3: fact [parent=Global]
    n 1

f4: fact [parent=Global]
    n 0
    Return value 1
```
Recursion in Environment Diagrams

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1 def fact(n):
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- The same function `fact` is called multiple times
- Different frames keep track of the different arguments in each call

(Demo)

```
Global frame

func fact(n) [parent=Global]

fact

f1: fact [parent=Global]
    n 3

f2: fact [parent=Global]
    n 2

f3: fact [parent=Global]
    n 1

f4: fact [parent=Global]
    n 0

Return value 1
```

Interactive Diagram
Recursion in Environment Diagrams

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1  def fact(n):
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- The same function `fact` is called multiple times.
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- What `n` evaluates to depends upon the current environment.

Interactive Diagram
Recursion in Environment Diagrams

The same function `fact` is called multiple times.
Different frames keep track of the different arguments in each call.
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```python
1  def fact(n):
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(Demo)

```
Global frame
-

f1: fact [parent=Global]  
   fact
   n 3

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   n 1

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   n 0
   Return value 1
```
Recursion in Environment Diagrams

1. The same function `fact` is called multiple times.
2. Different frames keep track of the different arguments in each call.
3. What `n` evaluates to depends upon the current environment.
4. Each call to `fact` solves a simpler problem than the last: smaller `n`.

```
1 def fact(n):
2     if n == 0:
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(Demo)

- Global frame
  
- `fact` frame
  - `n = 3`

- `f1` frame
  - `n = 2`

- `f2` frame
  - `n = 1`

- `f3` frame
  - `n = 0`

- Return value: 1

**Interactive Diagram**
Iteration vs Recursion
Iteration vs Recursion

Iteration is a special case of recursion
Iteration vs Recursion

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\[ 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 \]
Iteration vs Recursion

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Using while:
Iteration vs Recursion

Iteration is a special case of recursion

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Using while:

```python
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```
Iteration vs Recursion

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4! = 4 · 3 · 2 · 1 = 24

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Using recursion:

```python
def fact(n):
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Math:
Iteration vs Recursion

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Math:

\[ n! = \prod_{k=1}^{n} k \]
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\[ n! = \begin{cases} 
1 & \text{if } n = 0 \\
 n \cdot (n - 1)! & \text{otherwise}
\end{cases} \]
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Names: n, total, k, fact_iter

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\[ n! = \begin{cases} 
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Names: \( n, \text{fact} \)
Verifying Recursive Functions
The Recursive Leap of Faith
The Recursive Leap of Faith

Photo by Kevin Lee, Preikestolen, Norway
The Recursive Leap of Faith

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The Recursive Leap of Faith

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Is fact implemented correctly?
The Recursive Leap of Faith

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Is fact implemented correctly?

1. Verify the base case
The Recursive Leap of Faith

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Is fact implemented correctly?

1. Verify the base case

2. Treat `fact` as a functional abstraction!

Photo by Kevin Lee, Preikestolen, Norway
The Recursive Leap of Faith

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Is fact implemented correctly?

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2. Treat `fact` as a functional abstraction!
3. Assume that `fact(n-1)` is correct
The Recursive Leap of Faith

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Is fact implemented correctly?

1. Verify the base case

2. Treat `fact` as a functional abstraction!

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4. Verify that `fact(n)` is correct
Mutual Recursion
The Luhn Algorithm
The Luhn Algorithm

Used to verify credit card numbers
The Luhn Algorithm

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The Luhn Algorithm

Used to verify credit card numbers


- **First:** From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., $7 \times 2 = 14$), then sum the digits of the products (e.g., 10: $1 + 0 = 1$, 14: $1 + 4 = 5$)
The Luhn Algorithm

Used to verify credit card numbers


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• **Second:** Take the sum of all the digits
The Luhn Algorithm

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```
  1  3  8  7  4  3
```
The Luhn Algorithm

Used to verify credit card numbers


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<p>| | | | | | |</p>
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The Luhn Algorithm

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The Luhn Algorithm

Used to verify credit card numbers


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= 30

The Luhn sum of a valid credit card number is a multiple of 10
The Luhn Algorithm

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The Luhn sum of a valid credit card number is a multiple of 10 (Demo)
Recursion andIteration
Converting Recursion to Iteration
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Can be tricky: Iteration is a special case of recursion.
Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last

Converting Recursion to Iteration

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Converting Recursion to Iteration

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```python
def sum_digits(n):
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```

```
What's left to sum
```
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```

A partial sum

What's left to sum
Converting Recursion to Iteration

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    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last  # A partial sum
```

(Demo)
Converting Iteration to Recursion
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More formulaic: Iteration is a special case of recursion.
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Idea: The state of an iteration can be passed as arguments.
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```python
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum
```
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def sum_digits_rec(n, digit_sum):
    if n == 0:
        return digit_sum
    else:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
```
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```

Updates via assignment become...
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```

Updates via assignment become...

...arguments to a recursive call