Tree Recursion
Announcements
Order of Recursive Calls
The Cascade Function

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```python
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

• If two implementations are equally clear, then shorter is usually better
• In this case, the longer implementation is more clear (at least to me)
• When learning to write recursive functions, put the base cases first
• Both are recursive functions, even though only the first has typical structure
Example: Inverse Cascade
Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
grow(n)
print(n)
shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow = lambda n: f_then_g(
shrink = lambda n: f_then_g(
```
Tree Recursion
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more
than one recursive call

\[
\begin{align*}
  n: & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
  \text{fib}(n): & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
\end{align*}
\]

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

A Tree-Recursive Process

The computational process of fib evolves into a tree structure
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.

(We will speed up this computation dramatically in a few weeks by remembering results)
Example: Counting Partitions
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

\[
\text{count_partitions}(6, 4)
\]

2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6
The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - \(\text{count_partitions}(2, 4)\)
  - \(\text{count_partitions}(6, 3)\)
- Tree recursion often involves exploring different choices.
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
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- Solve two simpler problems:
  - $\text{count_partitions}(2, 4)$
  - $\text{count_partitions}(6, 3)$
- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m

result = count_partitions(1, 4) + count_partitions(2, 4) + count_partitions(3, 3) + count_partitions(6, 3)
```

(Demo)