Tree Recursion
Announcements
Order of Recursive Calls
The Cascade Function

(Demo)
The Cascade Function

def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
The Cascade Function

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```

Program output:

```
123
12
1
12
```
The Cascade Function

```
1 def cascade(n):
2     if n < 10:
3         print(n)
4     else:
5         print(n)
6         cascade(n//10)
7     print(n)
8
cascade(123)
```

(Demo)

- Each cascade frame is from a different call to cascade.

Program output:
```
123
12
1
12
```
The Cascade Function

1. **def cascade(n):**
   - if n < 10:
     - print(n)
   - else:
     - print(n)
   - cascade(n//10)
   - print(n)

2. **cascade(123)**

Program output:

```
123
12
1
12
```

(Demo)

- Each cascade frame is from a different call to `cascade`.
- Until the Return value appears, that call has not completed.
The Cascade Function

1. def cascade(n):
   2.     if n < 10:
   3.         print(n)
   4.     else:
   5.         print(n)
   6.         cascade(n//10)
   7.         print(n)
8.     cascade(123)

Program output:
123
12
1
12

(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.
The Cascade Function

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```

Program output:
```
123
12
1
12
```

(Demo)

- Each cascade frame is from a different call to `cascade`.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.
The Cascade Function

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```

(Demo)

- Each cascade frame is from a different call to `cascade`.
- Until the Return value appears, that call has not completed.
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Program output:
```
123
12
1
12
```
The Cascade Function

Program output:

```
123
12
1
12
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(Demo)

- Each cascade frame is from a different call to `cascade`.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```
The Cascade Function

• Each cascade frame is from a different call to cascade.
• Until the Return value appears, that call has not completed.
• Any statement can appear before or after the recursive call.

Program output:

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```
Two Definitions of Cascade

(Demo)
Two Definitions of Cascade

(Demo)

def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
Two Definitions of Cascade

(Demo)

def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)

• If two implementations are equally clear, then shorter is usually better
• In this case, the longer implementation is more clear (at least to me)
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
Two Definitions of Cascade

(Demo)

def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)

• If two implementations are equally clear, then shorter is usually better
• In this case, the longer implementation is more clear (at least to me)
• When learning to write recursive functions, put the base cases first
• Both are recursive functions, even though only the first has typical structure
Example: Inverse Cascade
Inverse Cascade

Write a function that prints an inverse cascade:
Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```
Inverse Cascade

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```
Inverse Cascade

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
grow(n)
print(n)
shrink(n)

def f_then_g(f, g, n):
if n:
    f(n)
    g(n)
```

**Inverse Cascade**

Write a function that prints an inverse cascade:

```python
1
grow = lambda n: f_then_g(grow, print, n // 10)
shrink = lambda n: f_then_g(print, shrink, n // 10)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```python
```
```
Inverse Cascade

Write a function that prints an inverse cascade:

def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow = lambda n: f_then_g(grow, print, n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
Tree Recursion
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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.
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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[ n: \ 0, 1, 2, 3, 4, 5, 6, 7, 8, \]

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

\[
\begin{align*}
n: & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \\
\text{fib}(n): & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21,
\end{align*}
\]

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

\[
\begin{align*}
n: & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
fib(n): & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21,
\end{align*}
\]

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

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\begin{align*}
\text{n:} & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
\text{fib(n):} & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
\end{align*}
\]
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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\begin{align*}
n & : \ 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
\text{fib(n)} & : \ 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
\end{align*}
\]

```python
def fib(n):
```

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{align*}
n & : 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
n & : 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465 \\
\text{def } \text{fib(n):} & \\
\text{if } n == 0: & 
\end{align*}
\]

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

```
def fib(n):
    if n == 0:
        return 0
```

n: 0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35

fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

\[
\begin{align*}
\text{n:} & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
\text{fib(n):} & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
\end{align*}
\]

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    fib(n-1) + fib(n-2)
```

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{align*}
n &: 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
\text{fib}(n) &: 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
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def fib(n):
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        return 1
```

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

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def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
```

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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\begin{align*}
n: & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
fib(n): & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465 
\end{align*}
\]

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

A Tree-Recursive Process

The computational process of fib evolves into a tree structure
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

\[ \text{fib}(5) \]
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

```
  fib(5)
    |
    |
  fib(3)
```
A Tree-Recursive Process

The computational process of fib evolves into a tree structure.
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A Tree-Recursive Process

The computational process of fib evolves into a tree structure.
Repetition in Tree-Recursive Computation
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This process is highly repetitive; fib is called on the same argument multiple times.
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Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times

(We will speed up this computation dramatically in a few weeks by remembering results)
Example: Counting Partitions
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

\[
\text{count_partitions}(6, 4)
\]
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

```
count_partitions(6, 4)
```

```
2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6
```
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

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\begin{align*}
2 + 4 & = 6 \\
1 + 1 + 4 & = 6 \\
3 + 3 & = 6 \\
1 + 2 + 3 & = 6 \\
1 + 1 + 1 + 3 & = 6 \\
2 + 2 + 2 & = 6 \\
1 + 1 + 2 + 2 & = 6 \\
1 + 1 + 1 + 1 + 2 & = 6 \\
1 + 1 + 1 + 1 + 1 + 1 & = 6 \\
\end{align*}
\]
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2 + 4 & = 6 \\
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$\text{count_partitions}(6, 4)$

2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
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1 + 1 + 1 + 1 + 1 + 1 = 6
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

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\text{count_partitions}(6, 4)
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Counting Partitions

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\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in non-decreasing order.

\[ \text{count_partitions}(6, 4) \]

- Recursive decomposition: finding simpler instances of the problem.
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Counting Partitions

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\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
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  - Don't use any 4
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

$$\text{count_partitions}(6, 4)$$

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - \text{count_partitions}(2, 4)
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - \( \text{count_partitions}(2, 4) \)
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

\[ \text{count_partitions}(6, 4) \]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - \( \text{count_partitions}(2, 4) \)
  - \( \text{count_partitions}(6, 3) \)
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

$count\_partitions(6, 4)$

- Recursive decomposition: finding simpler instances of the problem.
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- Solve two simpler problems:
  - count\_partitions(2, 4)
  - count\_partitions(6, 3)
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

$\text{count\_partitions}(6, 4)$

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
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  - Don't use any 4
- Solve two simpler problems:
  - $\text{count\_partitions}(2, 4)$
  - $\text{count\_partitions}(6, 3)$
- Tree recursion often involves exploring different choices.
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

```
count_partitions(6, 4)
```

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

• Recursive decomposition: finding simpler instances of the problem.

• Explore two possibilities:
  • Use at least one 4
  • Don't use any 4

• Solve two simpler problems:
  • count_partitions(2, 4)
  • count_partitions(6, 3)

• Tree recursion often involves exploring different choices.
The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in non-decreasing order.

\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
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Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

• Recursive decomposition: finding simpler instances of the problem.

• Explore two possibilities:
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• Solve two simpler problems:
  • count_partitions(2, 4)
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The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

- Recursive decomposition: finding simpler instances of the problem.

- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`

- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    # Your implementation goes here
```
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - \( \text{count_partitions}(2, 4) \)
  - \( \text{count_partitions}(6, 3) \)
- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if n < 0:
        return 0
    if n == 0:
        return 1
    else:
        # Use at least one 4
        first_case = count_partitions(n-4, m) if n >= 4 else 0
        # Don't use any 4
        second_case = count_partitions(n, m-1)
        return first_case + second_case
```
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

- Recursive decomposition: finding simpler instances of the problem.

- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`

- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if n < m:
        return 0
    elif n == m:
        return 1
    else:
        with_m = count_partitions(n-m, m)
        return count_partitions_without_4(n, m) + with_m
```
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
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  - \( \text{count_partitions}(2, 4) \)
  - \( \text{count_partitions}(6, 3) \)
- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if m > n:
        return 1
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

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def count_partitions(n, m):
    if m == 0:
        return 0
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        return with_m + without_m
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(Demo)