Sequences
Class outline:

- Box+Pointer
- Slicing
- Recursive exercises
- Built-ins for iterables
Where to ask questions?

- **Zoom chat**: Good if you like getting responses from classmates or the lecture helper.
- **Zoom Q&A**: Good for asking questions that likely interest most students, and that should be answered in lecture.
- **Post-lecture OH**: Good for recapping a topic that went too fast. Or any questions!
- **Piazza thread**: Good for longer questions, tangential questions, or any unanswered questions.
Box + Pointer
Lists in environment diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element.

```python
pair = [1, 2]
```

Try in PythonTutor.
Nested lists in environment diagrams

Each box either contains a primitive value or points to a compound value.

matrix = [ [1,2,0,4], [0,1,3,-1], [0,0,1,8] ]
Nested lists in environment diagrams

A very nested list:

```python
worst_list = [ [1, 2], [], [ [3, False, None], [4, lambda: 5]]]
```

View in PythonTutor
Slicing
Slicing syntax

Slicing a list creates a new list with a subsequence of the original list.

```
letters = ["A", "B", "C", "D", "E", "F"]
           #  0  1  2  3  4  5
sublist1 = letters[1:]
sublist2 = letters[1:4]
```

Slicing also works for strings.

```
compound_word = "cortaúñas"
word1 = compound_word[:5]
word2 = compound_word[5:]
```

Negatives indices and steps can also be specified.
Slicing syntax

Slicing a list creates a new list with a subsequence of the original list.

```python
letters = ["A", "B", "C", "D", "E", "F"]
    #  0   1   2   3   4   5
sublist1 = letters[1:]     # ['B', 'C', 'D', 'E', 'F']
sublist2 = letters[1:4]
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### Slicing syntax

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Slicing also works for strings.

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compound_word = "cortaúñas"

word1 = compound_word[:5]    # "corta"
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Negatives indices and steps can also be specified.
Slicing syntax

Slicing a list creates a new list with a subsequence of the original list.

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letters = ['A', 'B', 'C', 'D', 'E', 'F']
        #  0  1  2  3  4  5
sublist1 = letters[1:]     # ['B', 'C', 'D', 'E', 'F']
sublist2 = letters[1:4]     # ['B', 'C', 'D']
```

Slicing also works for strings.

```python
compound_word = "cortaúñas"
word1 = compound_word[:5]   # "corta"
word2 = compound_word[5:]   # "úñas"
```

Negatives indices and steps can also be specified.
Copying whole lists

Slicing a whole list copies a list:

```python
listA = [2, 3]
listB = listA

listC = listA[:]
listA[0] = 4
listB[1] = 5
```

**list()** creates a new list containing existing elements from any iterable:

```python
listA = [2, 3]
listB = listA

listC = list(listA)
listA[0] = 4
listB[1] = 5
```

Try both in PythonTutor.

Python3 provides more ways in the `copy` module.
Recursion exercises
Recursively sum a list

Let's code this up recursively:

```python
def sum_nums(nums):
    """Returns the sum of the numbers in NUMS."
    >>> sum_nums([6, 24, 1984])
    2014
    >>> sum_nums([-32, 0, 32])
    0
    """

Docstrings typically would not specify whether an approach was recursive or iterative, since that is an implementation detail.

However, we'll make it clear in assignments and exam questions.
Recursively sum a list (solution)

```python
def sum_nums(nums):
    """Returns the sum of the numbers in NUMS.
    >>> sum_nums([6, 24, 1984])
    2014
    >>> sum_nums([-32, 0, 32])
    0
    """
    if (nums == []):
        return 0
    else:
        return nums[0] + sum_nums( nums[1:] )
```

When recursively processing lists, the base case is often the empty list and the recursive case is often all-but-the-first items.
Iteratively sum a range

Let's code this up iteratively:

```python
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusive).
    """
    >>> sum_up_to(5)
    15
    """
```
Iteratively sum a range (solution)

Using the `range` type:

```python
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusive)."
    >>> sum_up_to(5)
    15
    """
    sum = 0
    for n in range(0, n+1):
        sum += n
    return sum
```

Remember that `range(start, end)` always ends right before `end`. 
Recursively sum a range

Now try it recursively:

```python
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusive)."
    >>> sum_up_to(5)
    15
    """
```
Recursively sum a range (solution)

Now try it recursively:

```python
def sum_up_to(n):
    """Returns the sum of positive numbers from 1 up to N (inclusive).""
    if n == 1:
        return 1
    else:
        return n + sum_up_to(n-1)

>>> sum_up_to(5)
15
"""
```
Reversing a string
Recursively reversing a string

```python
def reverse(s):
    """Returns a string with the letters of S in the inverse order.
    >>> reverse('ward')
    'draw'
    """
```

Breaking it down into subproblems:

```python
reverse("ward") = reverse("ard") = reverse("rd") = reverse("d") =
```
def reverse(s):
    '''Returns a string with the letters of S in the inverse order.
    >>> reverse('ward')
    'draw'
    '''

Breaking it down into subproblems:

reverse("ward") = reverse("ard") + "w"
reverse("ard") = reverse("rd") + "a"
reverse("rd") = reverse("d") + "r"
reverse("d") =
Recursively reversing a string

def reverse(s):
    """Returns a string with the letters of S in the inverse order.
    >>> reverse('ward')
    'draw'
    """
    reverse("ward") = reverse("ard") + "w"
    reverse("ard") = reverse("rd") + "a"
    reverse("rd") = reverse("d") + "r"
    reverse("d") = "d"
Recursively reversing a string (solution)

def reverse(s):
    """Returns a string with the letters of S in the inverse order.
    >>> reverse('ward')
    'draw'
    """
    if len(s) == 1:
        return s
    else:
        return reverse(s[1:]) + s[0]

When recursively processing strings, the base case is typically an empty string or single-character string, and the recursive case is often all-but-the-first characters.
Reversely reversing a string (visual)
Exercise: Reversing a number

def reverse(n):
    """Returns N with the digits reversed.
    >>> reverse_digits(123)
    321
    """

See walkthrough video here
Helper functions

If a recursive function needs to keep track of more state than the arguments of the original function, you may need a helper function.

def fUnKyCaSe(text):
    """Returns TEXT in fUnKyCaSe
    >>> fUnKyCaSe("wats up")
    'wAtS Up'
    """
Helper functions

If a recursive function needs to keep track of more state than the arguments of the original function, you may need a helper function.

```python
def fUnKyCaSe(text):
    """Returns TEXT in fUnKyCaSe
    >>> fUnKyCaSe("wats up")
    'wAtS Up'
    ""

def toggle_case(letter, should_up_case):
    return letter.upper() if should_up_case else letter.lower()

def up_down(text, should_up_case):
    if len(text) == 1:
        return toggle_case(text, should_up_case)
    else:
        return toggle_case(text[0], should_up_case) + up_down(text[1:], not should_up_case)

return up_down(text, False)
```
# Recursion on different data types

<table>
<thead>
<tr>
<th>Data type</th>
<th>Base case condition</th>
<th>Current item</th>
<th>Recursive case argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
<td>== 0</td>
<td>n % 10</td>
<td>n // 10</td>
</tr>
<tr>
<td></td>
<td>== 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lists</td>
<td>== []</td>
<td>L[0]</td>
<td>L[1:]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>L[::1]</td>
</tr>
<tr>
<td>Strings</td>
<td>== ''</td>
<td>S[0]</td>
<td>S[1:]</td>
</tr>
<tr>
<td></td>
<td>len(S) == 1</td>
<td></td>
<td>S[::1]</td>
</tr>
</tbody>
</table>
Built-in functions for iterables
Functions that process iterables

The following built-in functions work for sequence types (lists, strings, etc) and any other `iterable` data type.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>sum(iterable, start)</code></td>
<td>Returns the sum of values in <code>iterable</code>, initializing sum to <code>start</code></td>
</tr>
<tr>
<td><code>all(iterable)</code></td>
<td>Return True if all elements of <code>iterable</code> are true (or if <code>iterable</code> is empty)</td>
</tr>
<tr>
<td><code>any(iterable)</code></td>
<td>Return True if any element of <code>iterable</code> is true. Return False if <code>iterable</code> is empty.</td>
</tr>
<tr>
<td><code>max(iterable, key=None)</code></td>
<td>Return the max value in <code>iterable</code></td>
</tr>
<tr>
<td><code>min(iterable, key=None)</code></td>
<td>Return the min value in <code>iterable</code></td>
</tr>
</tbody>
</table>
Examples with sum/any/all

```
sum([73, 89, 74, 95], 0)  # 331

all([True, True, True, True])
any([False, False, False, True])

all([x < 5 for x in range(5)])

perfect_square = lambda x: x == round(x ** 0.5) ** 2
any([perfect_square(x) for x in range(50, 60)])
```
Examples with sum/any/all

```
sum([73, 89, 74, 95], 0)  # 331

all([True, True, True, True])  # True
any([False, False, False, True])

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```
Examples with sum/any/all

```python
sum([73, 89, 74, 95], 0)  # 331
```

```python
class True:
    any([True, True, True, True])  # True

all([[False, False, False, True]])  # True

all([x < 5 for x in range(5)])  # True
```

```python
perfect_square = lambda x: x == round(x ** 0.5) ** 2

any([perfect_square(x) for x in range(50, 60)])
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Examples with sum/any/all

sum([73, 89, 74, 95], 0)  # 331

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all([x < 5 for x in range(5)])  # True

perfect_square = lambda x: x == round(x ** 0.5) ** 2
any([perfect_square(x) for x in range(50, 60)])  # False
Examples with max/min

\[
\text{max}([73, 89, 74, 95]) \quad \# \quad 95 \\
\text{max}(["C+", "B+", "C", "A"]) \\
\text{max}(\text{range}(10))
\]
Examples with max/min

max([73, 89, 74, 95])  # 95
max(["C+", "B+", "C", "A"])  # C+
max(range(10))
Examples with max/min

```
max([73, 89, 74, 95])       # 95
max(['C+', 'B+', 'C', 'A'])  # C+
max(range(10))              # 9
```
Examples with max/min

\[
\text{max([73, 89, 74, 95])} \quad \# 95
\]
\[
\text{max(['C+', 'B+', 'C', 'A'])} \quad \# \text{C+}
\]
\[
\text{max(range(10))} \quad \# 9
\]

A key function can decide how to compare each value:

\[
\text{coords} = \text{[ [37, -144], [-22, -115], [56, -163] ]}
\]
\[
\text{max(coords, key=lambda coord: coord[0])}
\]
\[
\text{min(coords, key=lambda coord: coord[0])}
\]

\[
\text{gymnasts} = \text{[ ['Brittany', 9.15, 9.4, 9.3, 9.2],}
\quad \text{['Lea', 9, 8.8, 9.1, 9.5],}
\quad \text{['Maya', 9.2, 8.7, 9.2, 8.8] ]}
\]
\[
\text{min(gymnasts, key=lambda scores: min(scores[1:]))}
\]
\[
\text{max(gymnasts, key=lambda scores: sum(scores[1:], 0))}
\]
Examples with max/min

\[
\text{max}([73, 89, 74, 95]) \quad # \quad 95 \\
\text{max}(["C+", "B+", "C", "A"]) \quad # \quad C+ \\
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\text{gymnasts} = \text{[ ["Brittany", 9.15, 9.4, 9.3, 9.2],}
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Examples with max/min

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max([73, 89, 74, 95])  # 95
max(["C+", "B+", "C", "A"])  # C+
max(range(10))  # 9
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    ["Lea", 9, 8.8, 9.1, 9.5],
    ["Maya", 9.2, 8.7, 9.2, 8.8] ]
min(gymnasts, key=lambda scores: min(scores[1:]的思想))  # ["Maya", ..
max(gymnasts, key=lambda scores: sum(scores[1:], 0))  # ["Brittany"
```
Python Project of The Day!
Sea Level Rise

Sea Level Rise, by Douwe Osinga: Visualize sea levels and population density on interactive maps.

Technologies used: Python (notebook) with PIL/numpy/Rasterio, HTML/CSS/JS with PanZoom
(Github repository)