Data Abstraction
Announcements
Data Abstraction
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Data Abstraction

- Compound values combine other values together
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  - A date: a year, a month, and a day
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  ▪ A date: a year, a month, and a day
  ▪ A geographic position: latitude and longitude
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- Data abstraction lets us manipulate compound values as units
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  ▪ How data are represented (as parts)
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- Data abstraction: A methodology by which functions enforce an abstraction barrier between *representation* and *use*
Rational Numbers
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}} \quad \text{denominator}
\]
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}} \quad \text{denominator}
\]

Exact representation of fractions
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions
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As soon as division occurs, the exact representation may be lost! (Demo)
Assume we can compose and decompose rational numbers:
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \(\text{rational}(n, d)\) returns a rational number \(x\)
Rational Numbers

\[
\begin{array}{c}
\text{numerator} \\
\hline \\
\text{denominator}
\end{array}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \( \text{rational}(n, d) \) returns a rational number \( x \)
- \( \text{numer}(x) \) returns the numerator of \( x \)
Rational Numbers

exact representation of fractions
A pair of integers
As soon as division occurs, the exact representation may be lost! (Demo)
Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
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Rational Numbers

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number $x$
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Rational Numbers

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- \text{rational}(n, d): \text{return} \text{ a rational number } x
- \text{numer}(x): \text{return the numerator of } x
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Rational Number Arithmetic

Example

General Form
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5}
\]

General Form
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

Example

General Form
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy}
\]
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

\[
\frac{3}{2} + \frac{3}{5}
\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

\[ \frac{3}{2} \times \frac{3}{5} = \frac{9}{10} \]

\[ \frac{3}{2} + \frac{3}{5} = \frac{21}{10} \]

Example

General Form

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Rational Number Arithmetic

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Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

- `rational(n, d)` returns a rational number \( x \)
- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

- `rational(n, d)` returns a rational number \( \frac{n}{d} \)
- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

\[
\begin{align*}
\frac{nx}{dx} \times \frac{ny}{dy} &= \frac{nx \times ny}{dx \times dy} \\
\frac{nx}{dx} + \frac{ny}{dy} &= \frac{nx \times dy + ny \times dx}{dx \times dy}
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• denom(x) returns the denominator of x
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Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))

def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

- `rational(n, d)` returns a rational number \( \frac{n}{d} \)
- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

These functions implement an abstract representation for rational numbers.
Rational Number Arithmetic Implementation

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def mul_rational(x, y):
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    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

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    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rations_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- `rational(n, d)` returns a rational number \( \frac{n}{d} \)
- `numer(x)` returns the numerator of \( x \)
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These functions implement an abstract representation for rational numbers.
Pairs
Representing Pairs Using Lists
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```python
>>> pair = [1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:

Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal: Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

- A list literal: Comma-separated expressions in brackets
- "Unpacking" a list
- Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]  # A list literal: Comma-separated expressions in brackets
>>> pair
[1, 2]

>>> x, y = pair  # "Unpacking" a list
>>> x
1
>>> y
2

>>> pair[0]  # Element selection using the selection operator
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]  # A list literal:
>>> pair
[1, 2]

>>> x, y = pair  # "Unpacking" a list
>>> x
1
>>> y
2

>>> pair[0]  # Element selection using the selection operator
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

Construct a list
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

```python
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
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def numer(x):
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Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

(Demo)
Reducing to Lowest Terms

Example:
Reducing to Lowest Terms

Example:

\[
\begin{array}{c}
3 \\
\hline
2
\end{array} \quad \times \quad \begin{array}{c}
5 \\
\hline
3
\end{array}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{15}{6} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

$$\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}$$
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} + \frac{2}{5} + \frac{1}{10}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reduction to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
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\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2} \quad \quad \frac{25}{50} \times \frac{1}{25} = \frac{1}{2}
\]

from fractions import gcd
from fractions import gcd

def rational(n, d):

Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]

\[
\frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""

from fractions import gcd
def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \quad \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \quad \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]

**from fractions import gcd**

**def rational(n, d):**

"""Construct a rational that represents n/d in lowest terms."""

g = gcd(n, d)
return [n//g, d//g]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \ast \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \ast \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \ast \frac{1/25}{1/25} = \frac{1}{2}
\]

```python
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return [n//g, d//g]
```
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
\frac{15}{6} \times \frac{1/3}{1/3} &= \frac{5}{2} \\
\frac{25}{50} \times \frac{1/25}{1/25} &= \frac{1}{2}
\end{align*}
\]

from fractions import gcd

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def rational(n, d):
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    g = gcd(n, d)
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```

(Demo)
Abstraction Barriers
Abstraction Barriers
### Abstraction Barriers

<table>
<thead>
<tr>
<th>Parts of the program that...</th>
<th>Treat rationals as...</th>
<th>Using...</th>
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Abstraction Barriers

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Create rationals or implement rational operations
### Abstraction Barriers

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**Implementation of lists**
add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
Violating Abstraction Barriers

Does not use constructors

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(Demo)
Rationals Implemented as Functions
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Dictionaries

{"Dem": 0}
Limitations on Dictionaries
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If you want to associate multiple values with a key, store them all in a sequence value.