Data Abstraction

Announcements

Data Abstraction

Data Abstraction

- Compound values combine other values together
  - A date: a year, a month, and a day
  - A geographic position: latitude and longitude
- Data abstraction lets us manipulate compound values as units
- Isolate two parts of any program that uses data:
  - How data are represented (as parts)
  - How data are manipulated (as units)
- Data abstraction: A methodology by which functions enforce an abstraction barrier between ***representation*** and ***use***

All Programmers

Great Programmers

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor: `rational(n, d)` returns a rational number x

Selectors:
- `numer(x)` returns the numerator of x
- `denom(x)` returns the denominator of x

Rational Number Arithmetic

Example

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

General Form

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor: `rational`; Selectors: `numer`, `denom`

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

- `rational(n, d)` returns a rational number x
- `numer(x)` returns the numerator of x
- `denom(x)` returns the denominator of x

These functions implement an abstract representation for rational numbers

Representing Rational Numbers

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
2
```

Element selection using the selection operator

Representing Rational Numbers

```
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2} \qquad \frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from math import gcd
```

Greatest common divisor

```
def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return [n//g, d//g]
```

(Demo)

Abstraction Barriers

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational
Create rationals or implement rational operations	numerators and denominators	rational, numer, denom
Implement selectors and constructor for rationals	two-element lists	list literals and element selection

Implementation of lists

Violating Abstraction Barriers

Does not use constructors

Twice!

```
add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
```

No selectors!

And no constructor!

Data Representations

What are Data?

- We need to guarantee that constructor and selector functions work together to specify the right behavior
- Behavior condition: If we construct rational number x from numerator n and denominator d, then numer(x)/denom(x) must equal n/d
- Data abstraction uses selectors and constructors to define behavior
- If behavior conditions are met, then the representation is valid

You can recognize an abstract data representation by its behavior

(Demo)

Rationals Implemented as Functions

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

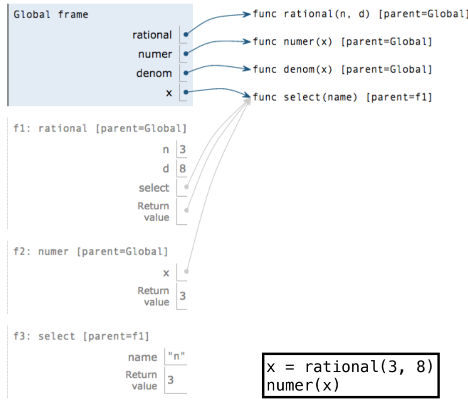
This function represents a rational number

Constructor is a higher-order function

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Selector calls x




```
x = rational(3, 8)
numer(x)
```

pythontutor.com/composingprograms.html#code=def%20rational%28n,
%20d%29%3A%0A%20%20%20%20def%20select%28name%29%3A%0A%20%20%20%20%20%20%20%20if%20name%20%3D%3D%20'n'%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20n%0A%20%20%20%20%20%20%20%20elif%20name%20%3D%3D%20'd'%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20d%0A%20%20%20%20return%20select%
0A%20%20%20%20%0Adef%20numer%28x%29%3A%0A%20%20%20%20return%20x%28'n'%29%0A%0Adef%20denom%28x%29%3A%0A%20%20%20%20return%20x%28'd'%29%0A%20%20%20%20%0Ax%20%3D%20rational%283,%208%29%0Anumer%28x%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0