Data Abstraction
Announcements
Data Abstraction
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• Compound values combine other values together
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  - A date: a year, a month, and a day
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• Compound values combine other values together
  - A date: a year, a month, and a day
  - A geographic position: latitude and longitude
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• Data abstraction lets us manipulate compound values as units
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• Isolate two parts of any program that uses data:
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  ▪ How data are represented (as parts)
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- Data abstraction: A methodology by which functions enforce an abstraction barrier between representation and use
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• Data abstraction: A methodology by which functions enforce an abstraction barrier between representation and use
Rational Numbers
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions
Rational Numbers

\[
\begin{array}{c}
\text{numerator} \\
\hline
\text{denominator}
\end{array}
\]

Exact representation of fractions

A pair of integers
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)
Rational Numbers

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\frac{\text{numerator}}{\text{denominator}}
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Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
- \text{numer}(x) \text{ returns the numerator of } x
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Numbers

Exact representation of fractions
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As soon as division occurs, the exact representation may be lost! (Demo)
Assume we can compose and decompose rational numbers:

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}} \quad \text{denominator}
\]

\text{rational}(n, d) \text{ returns a rational number } x

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Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Number Arithmetic

Example

General Form
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5}
\]

Example

General Form
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]
Rational Number Arithmetic

Example

\[ \frac{3}{2} \times \frac{3}{5} = \frac{9}{10} \]

General Form

\[ \frac{nx}{dx} \times \frac{ny}{dy} \]
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

\[ \frac{3}{2} \times \frac{3}{5} = \frac{9}{10} \]

\[ \frac{3}{2} + \frac{3}{5} \]

**Example**

**General Form**

\[ \frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy} \]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

\[
\frac{3}{2} + \frac{3}{5} = \frac{21}{10}
\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
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Rational Number Arithmetic

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\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
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General Form
Rational Number Arithmetic

Example

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\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
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\frac{3}{2} + \frac{3}{5} = \frac{21}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x
Rational Number Arithmetic Implementation

def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
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Rational Number Arithmetic Implementation

`def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))`

 Constructor

- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`
- `denom(x)` returns the denominator of `x`
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
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```

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```
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))
```

These functions implement an abstract representation for rational numbers.
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))

def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

- `rational(n, d)` returns a rational number $x$
- `numer(x)` returns the numerator of $x$
- `denom(x)` returns the denominator of $x$

These functions implement an abstract representation for rational numbers:

$$\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}$$
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))

def add_rational(x, y):
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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

- rational(n, d) returns a rational number x
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These functions implement an abstract representation for rational numbers.
Rational Number Arithmetic Implementation

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def mul_rational(x, y):
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    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- `rational(n, d)` returns a rational number \(x\)
- `numer(x)` returns the numerator of \(x\)
- `denom(x)` returns the denominator of \(x\)

These functions implement an abstract representation for rational numbers.
Representing Rational Numbers
Representing Pairs Using Lists
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]  # A list literal:
>>> pair
[1, 2]  # Comma-separated expressions in brackets

>>> x, y = pair
```
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```plaintext
>>> pair = [1, 2]  # A list literal:
>>> pair
[1, 2]  # Comma-separated expressions in brackets

>>> x, y = pair
>>> x
1
>>> y
2
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal: Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
Representing Rational Numbers

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    return [n, d]
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```
Representing Rational Numbers

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def rational(n, d):
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    return x[1]
```

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def rational(n, d):
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def numer(x):
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    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

(Demo)
Reducing to Lowest Terms

Example:
Reducing to Lowest Terms

Example:

\[
\begin{array}{c}
\frac{3}{2} \times \frac{5}{3}
\end{array}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} + \frac{2}{5} + \frac{1}{10}
\]

\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
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Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1}{25} = \frac{1}{2}
\]

`from math import gcd`
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
\frac{15}{6} \times \frac{1}{3} &= \frac{5}{2} \\
\frac{25}{50} \times \frac{1}{25} &= \frac{1}{2}
\end{align*}
\]

```
from math import gcd

def rational(n, d):
```

from math import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""

Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \\
\frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
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\end{align*}
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\frac{15}{6} \times \frac{1/3}{1/3} &= \frac{5}{2} \\
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\end{align*}
\]

```python
from math import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms.""
    g = gcd(n, d)
```
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &= \frac{1}{2}
\end{align*}
\]

```
from math import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return [n//g, d//g]
```
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \text{and} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2} \quad \text{and} \quad \frac{25}{50} \times \frac{1}{25} = \frac{1}{2}
\]

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Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
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\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
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```

(Demo)
Abstraction Barriers
Abstraction Barriers
## Abstraction Barriers

<table>
<thead>
<tr>
<th>Parts of the program that...</th>
<th>Treat rationals as...</th>
<th>Using...</th>
</tr>
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Abstraction Barriers

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Create rationals or implement rational operations
### Abstraction Barriers

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*Implementation of lists*
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*Implementation of lists*
add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
Violating Abstraction Barriers

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No selectors!
Violating Abstraction Barriers

Does not use constructors

Twice!

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No selectors!

And no constructor!
Violating Abstraction Barriers
Data Representations
What are Data?
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* We need to guarantee that constructor and selector functions work together to specify the right behavior.
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* Behavior condition: If we construct rational number \( x \) from numerator \( n \) and denominator \( d \), then \( \text{numer}(x)/\text{denom}(x) \) must equal \( n/d \).
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(Demo)
Rationals Implemented as Functions
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select

def numer(x):
    return x('n')

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Constructor is a higher-order function.

Selector calls `x`.
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This function represents a rational number

Constructor is a higher-order function

Selector calls x

x = rational(3, 8)
numer(x)
This function represents a rational number

Constructor is a higher-order function

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