Trees
Class outline:

- Hog winners
- Trees
Hog winners
Hog strategy contest

hog-contest.cs61a.org
Hog strategy contest

hog-contest.cs61a.org

At first, there was a 3-way tie for first: Nishant Bhakar, Toby Worledge, Asrith Devalaraju & Aayush Gupta
Hog strategy contest

hog-contest.cs61a.org

At first, there was a 3-way tie for first:
Nishant Bhakar, Toby Worledge, Asrith Devalaraju &
Aayush Gupta

Then we fixed a bug...
1) Nishant Bhakar, 2) Toby Worledge, 3) Jiayin Lin & Roger Yu
Hog dice contest

dice.cs61a.org

Much ❤️ for all the entries!

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<td>based on our true story</td>
<td>Bella Lee, Dayeon Jang</td>
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Trees
Trees

Recursive description

- A tree has a root label and a list of branches
- Each branch is itself a tree
- A tree with zero branches is called a leaf
- A tree starts at the root
Recursive description

- A tree has a root label and a list of branches
- Each branch is itself a tree
- A tree with zero branches is called a leaf
- A tree starts at the root

Relative description

- Each location in a tree is called a node
- Each node has a label that can be any value
- One node can be the parent/child of another
- The top node is the root node
Trees: Data abstraction

We want this constructor and selectors:

<table>
<thead>
<tr>
<th>constructor</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>tree(label, branches)</td>
<td>Returns a tree with root label and list of branches</td>
</tr>
<tr>
<td>label(tree)</td>
<td>Returns the root label of tree</td>
</tr>
<tr>
<td>branches(tree)</td>
<td>Returns the branches of tree (each a tree).</td>
</tr>
<tr>
<td>is_leaf(tree)</td>
<td>Returns true if tree is a leaf node.</td>
</tr>
</tbody>
</table>

```python
t = tree(3, [
    tree(1),
    tree(2, [
        tree(1),
        tree(1)
    ])
])

label(t)  # 3
is_leaf(branches(t)[0])  # True
```
Tree: Our implementation

```
t = tree(3, [
    tree(1),
    tree(2, [
        tree(1),
        tree(1)
    ])
])
```

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(tree):
    return tree[0]
```

Each tree is stored as a list where first element is label and subsequent elements are branches.

```
[3, [1], [2, [1], [1]]]
```
def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return len(branches(tree)) == 0
Tree processing

A tree is a recursive structure.

Each tree has:

- A label
- 0 or more branches, each a tree

Recursive structure implies recursive algorithm!
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if

    else:

What's the base case? What's the recursive call?
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        
    else:

What's the base case? What's the recursive call?
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:

What's the base case? What's the recursive call?
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if is_leaf(t):
        return 1
    else:
        leaves_under = 0
        for b in branches(t):
            leaves_under += count_leaves(b)
        return leaves_under

What's the base case? What's the recursive call?
Tree processing: Counting leaves

The `sum()` function sums up the items of an iterable.

```python
sum([1, 1, 1, 1])   # 4
```
Tree processing: Counting leaves

The `sum()` function sums up the items of an iterable.

```
sum([1, 1, 1, 1])  # 4
```

That leads to this shorter function:

```python
def count_leaves(t):
    """Returns the number of leaf nodes in T.""
    if is_leaf(t):
        return 1
    else:
        branch_counts = [count_leaves(b) for b in branches(t)]
        return sum(branch_counts)
```
Creating trees

A function that creates a tree from another tree is also often recursive.
Creating trees: Doubling labels

def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if

    else:

What's the base case? What's the recursive call?
def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if is_leaf(t):
        return [2 * label for label in t]
    else:
        return [double(child) for child in t]

What's the base case? What's the recursive call?
Creating trees: Doubling labels

def double(t):
    """Returns a tree identical to T, but with all labels doubled."""
    if is_leaf(t):
        return tree(label(t) * 2)
    else:

What's the base case? What's the recursive call?
def double(t):
    
    """Returns a tree identical to T, but with all labels doubled."

    if is_leaf(t):
        return tree(label(t) * 2)

    else:
        return tree(label(t) * 2, [double(b) for b in branches(t)])

What's the base case? What's the recursive call?
Creating trees: Doubling labels

A shorter solution:

```python
def double(t):
    """Returns the number of leaf nodes in T.""
    return tree(label(t) * 2,
               [double(b) for b in branches(t)])
```

Explicit base cases aren't always necessary in the final code, but it's useful to think in terms of base case vs. recursive case when learning.
Exercise: Printing trees

def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = tree(3, [tree(1), tree(2, [tree(1), tree(1)])])
    >>> print(t)
    3
    1
    2
    1
    1
    """
def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = tree(3, [tree(1), tree(2, [tree(1), tree(1)])])
    >>> print(t)
    3
     1
     2
     1
    1
    """
    print(indent * " " + label(t))
    for b in branches(t):  
        print_tree(t, indent + 2)
Exercise: List of leaves

```python
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = tree(20, [tree(12, [tree(9, [tree(7), tree(2)]), tree(3)]), tree(8, [tree(4), tree(4)])]
    >>> leaves(t)
    [7, 2, 3, 4, 4]
    """
```

Hint: If you sum a list of lists, you get a list containing the elements of those lists. The sum function takes a second argument, the starting value of the sum.

```python
sum([[1], [2, 3], [4], []]) # [1, 2, 3, 4]
sum([[1]], []) # [1]
sum([[[1]], [2]], []) # [[1], 2]
```
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = tree(20, [tree(12, [tree(9, [tree(7), tree(2)]), tree(8, [tree(4), tree(4)])]), tree(3)])
    >>> leaves(t)
    [7, 2, 3, 4, 4]
    """
    if is_leaf(t):
        return [label(t)]
    else:
        leaf_labels = [leaves(b) for b in branches(t)]
        return sum(leaf_labels, [])

Exercise: List of leaves (Solution)
def count_paths(t, total):
    """Return the number of paths from the root to any node in t for which the labels along the path sum to total."

>>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])]
>>> count_paths(t, 3)
2
>>> count_paths(t, 4)
2
>>> count_paths(t, 5)
0
>>> count_paths(t, 6)
1
>>> count_paths(t, 7)
2
"""
def count_paths(t, total):
    """Return the number of paths from the root to any node in t for which the labels along the path sum to total."

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = 1
    else:
        found = 0
    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
# Tree: Layers of abstraction

<table>
<thead>
<tr>
<th>Primitive Representation</th>
<th>1 2 3 &quot;a&quot; &quot;b&quot; &quot;c&quot;</th>
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<tr>
<td>Data abstraction</td>
<td>tree() branches() label()</td>
</tr>
<tr>
<td></td>
<td>is_leaf()</td>
</tr>
<tr>
<td>User program</td>
<td>double(t) count_leaves(t)</td>
</tr>
</tbody>
</table>

Each layer only uses the layer above it.
Trees, trees, everywhere!
Directory structures
Parse trees

For natural languages...

```
S
 /  \
/    \
NP   VP
 /  \
/    \
D    N  V    NP
 |    |  |    |
A    mouse eats a  cat.
```

Key: S = Sentence, NP = Noun phrase, D = Determiner, N = Noun, V = Verb, VP = Verb Phrase
Parse trees

For programming languages, too...

Key: E = expression
Python Project of The Day!
Kolibri

Kolibri: An open-source learning platform optimized for offline access.

Technologies used: Python, Django. (Github repository)