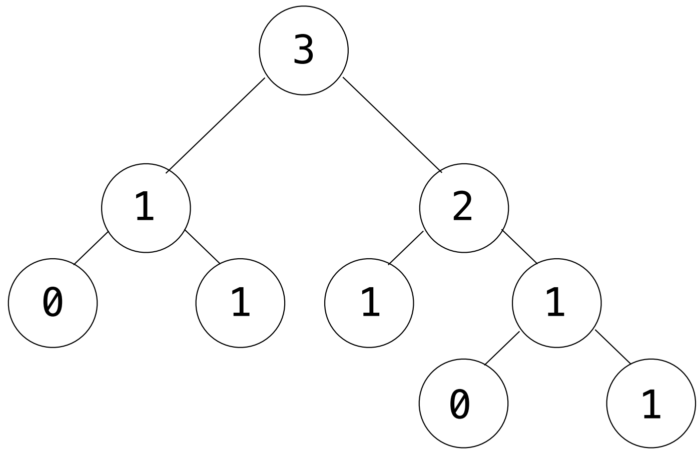
Trees

Announcements

Trees

Tree Abstraction



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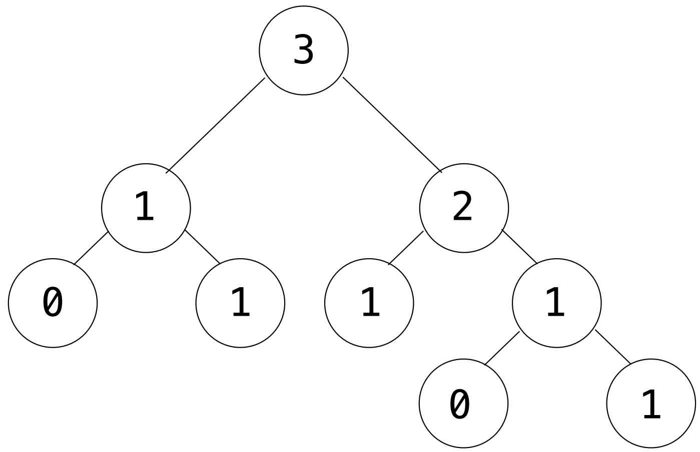

Recursive description **(wooden trees):**

Relative description **(family trees):**

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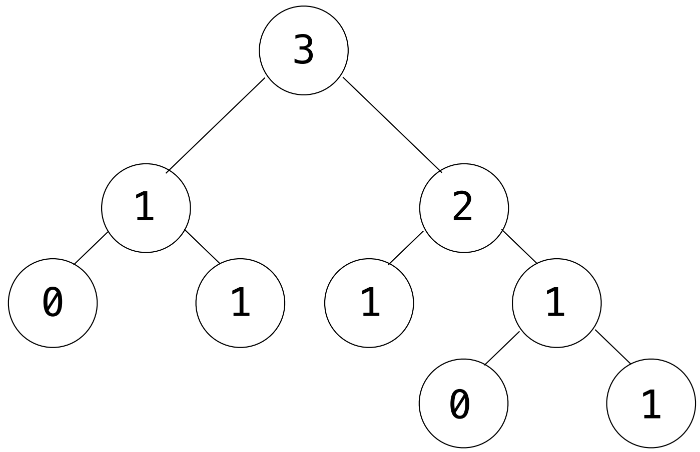

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A **tree** has a **root label** and a list of **branches**

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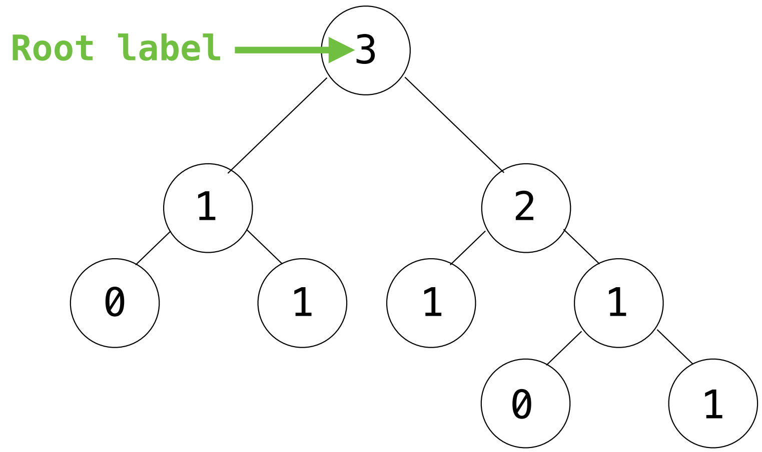

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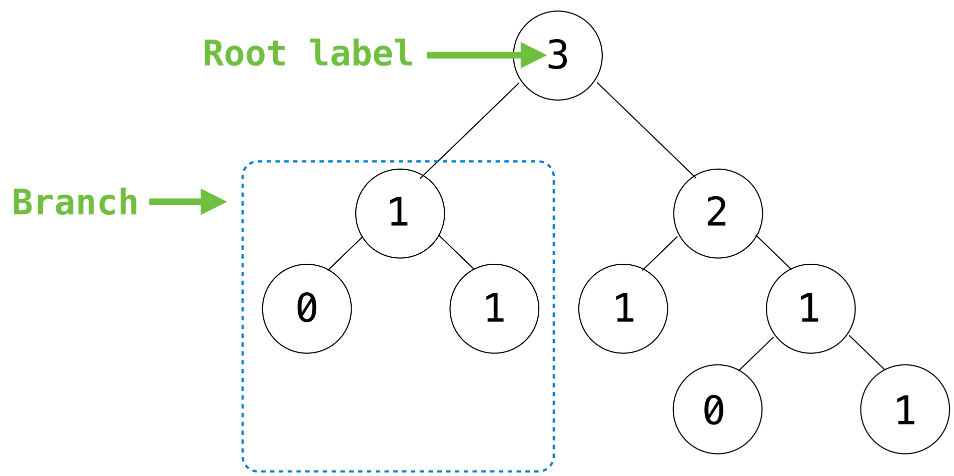

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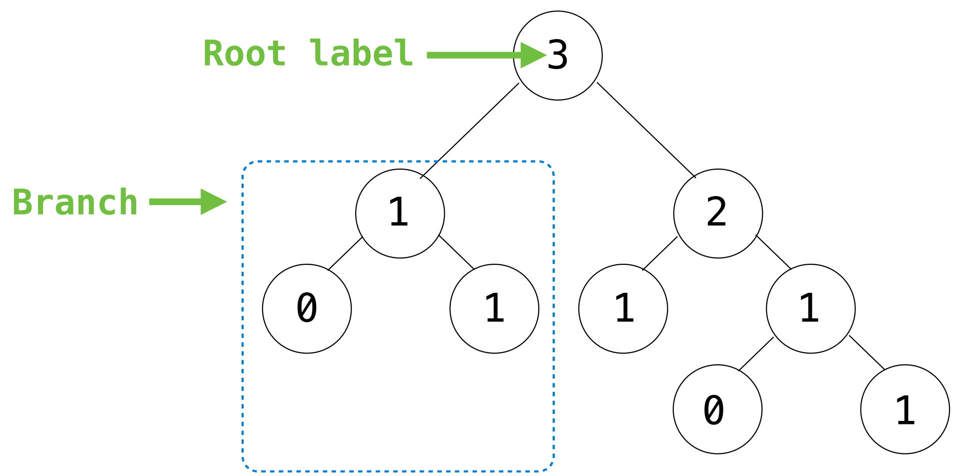

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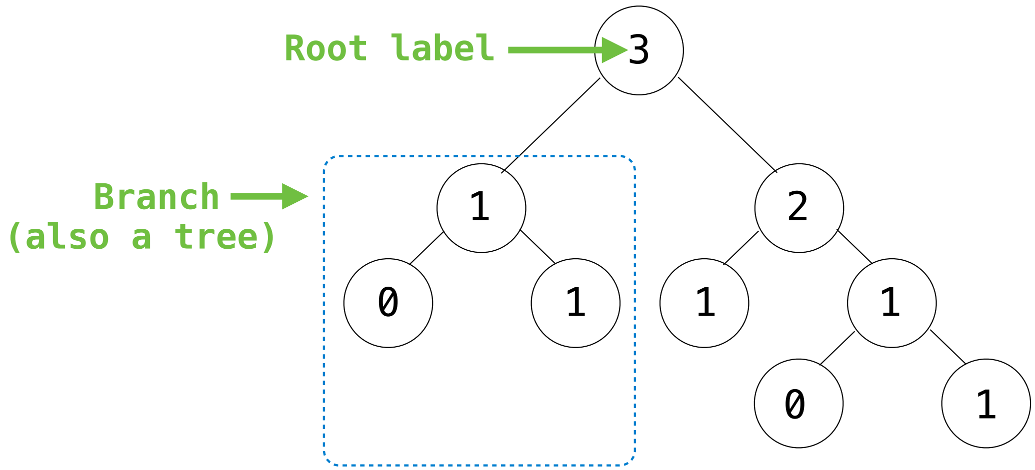

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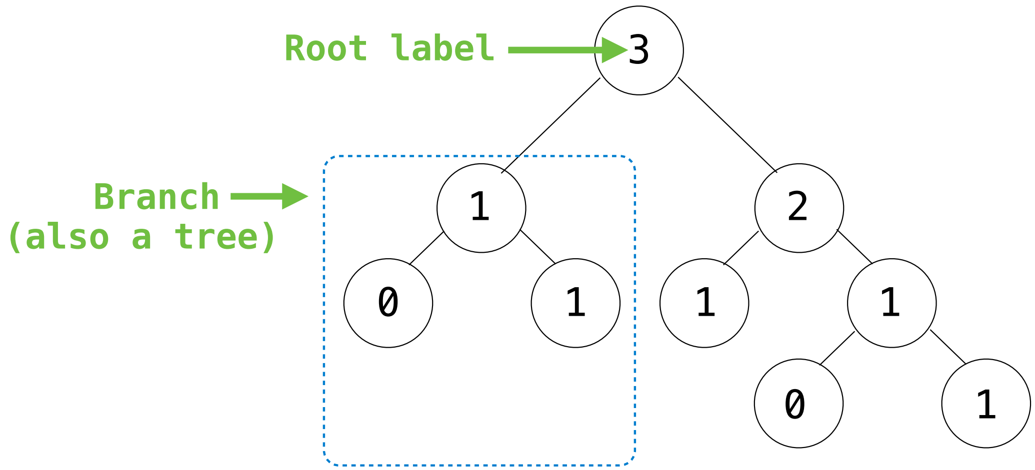

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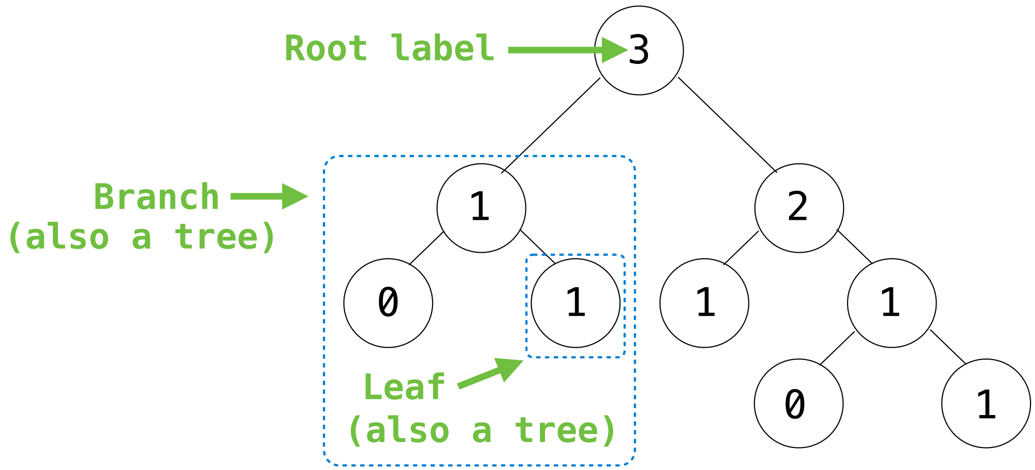

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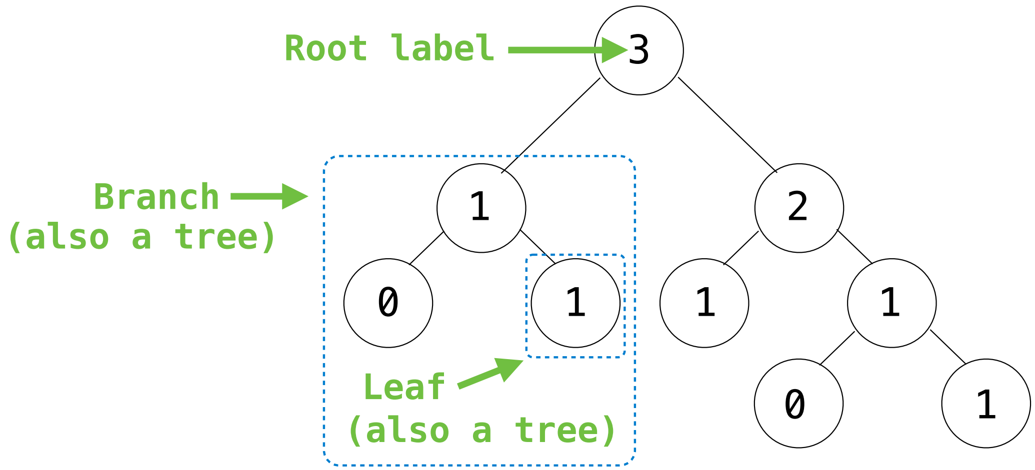

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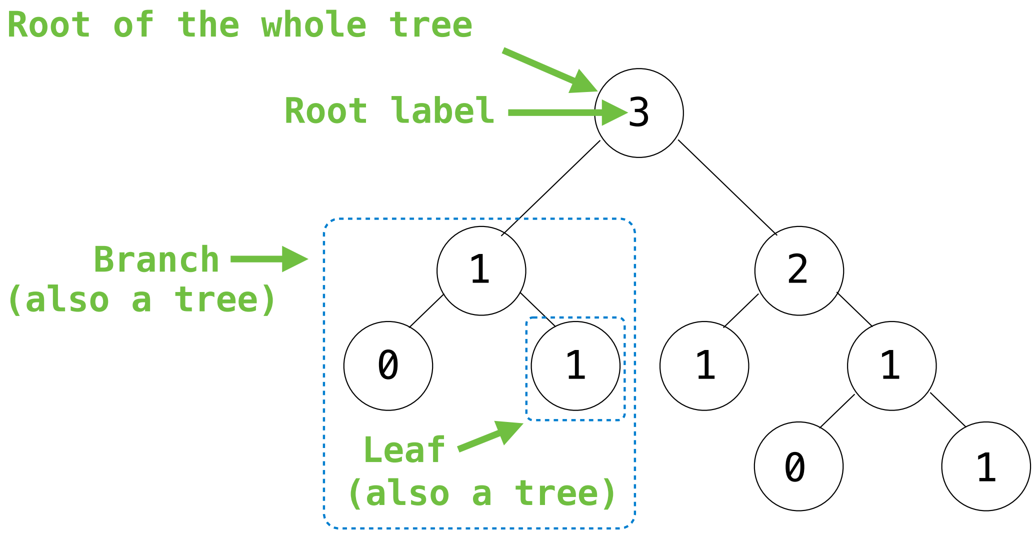

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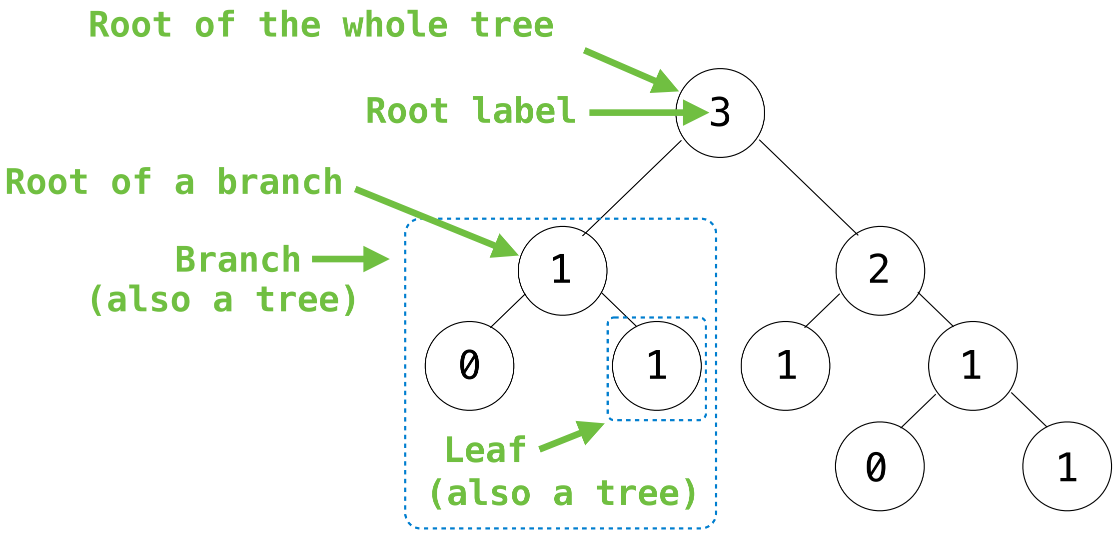

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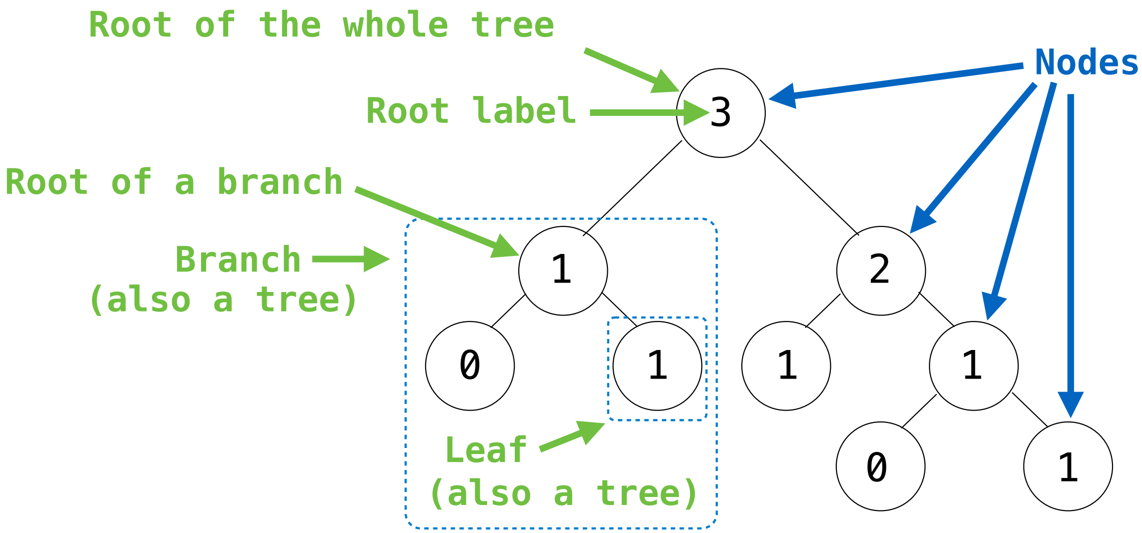

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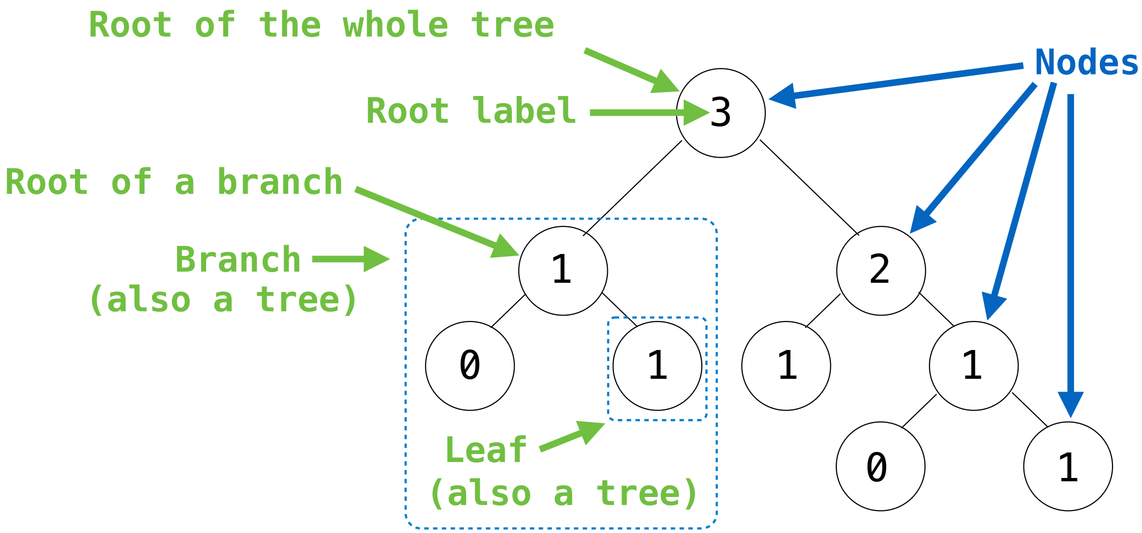

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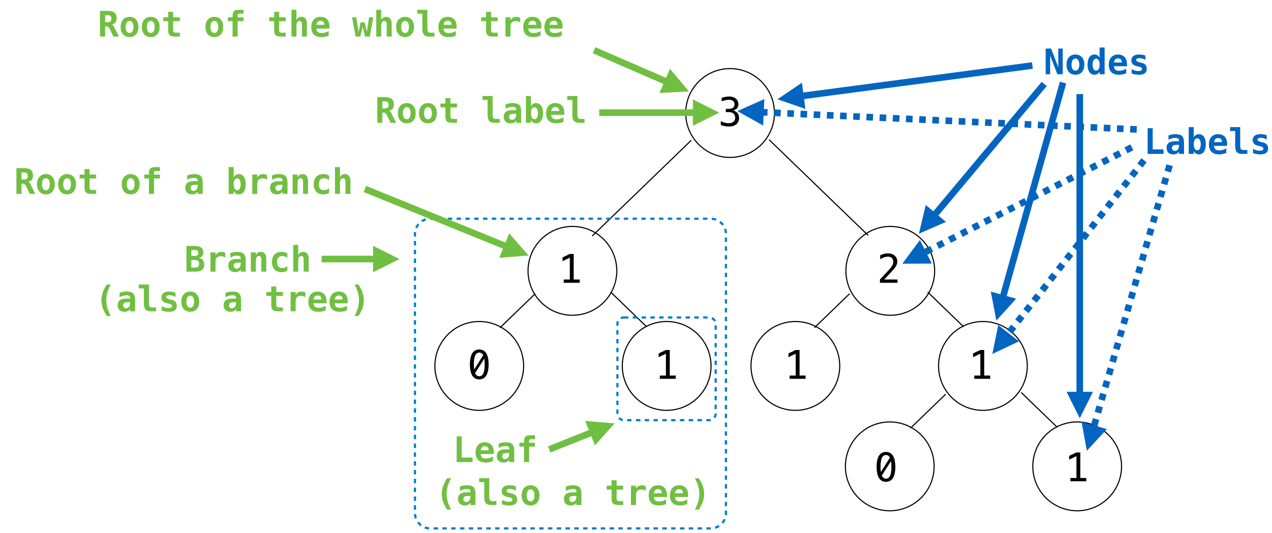

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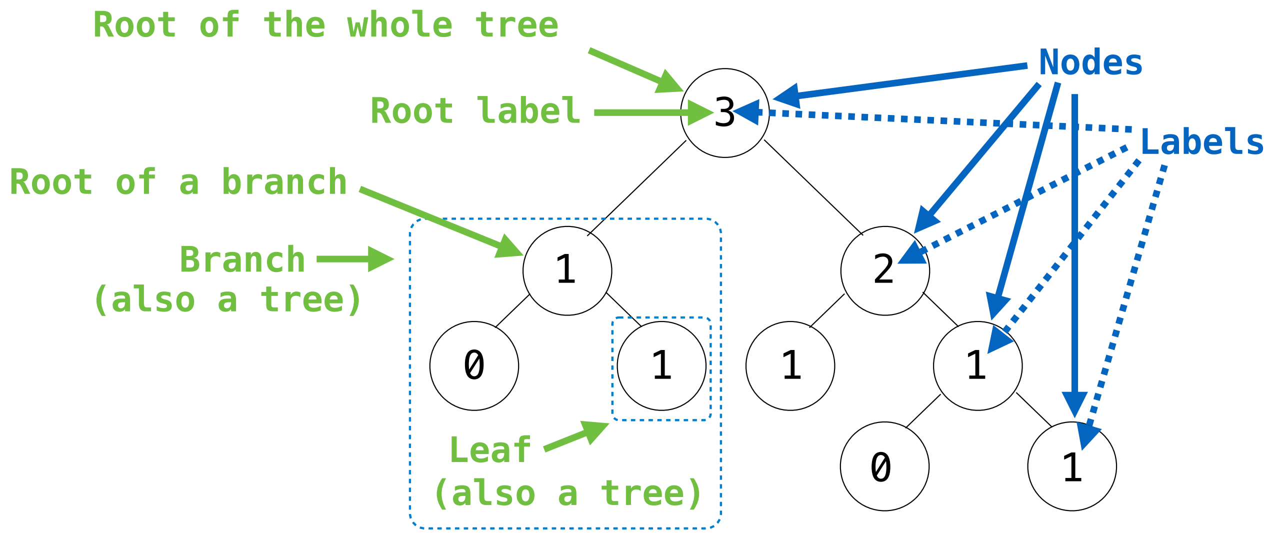

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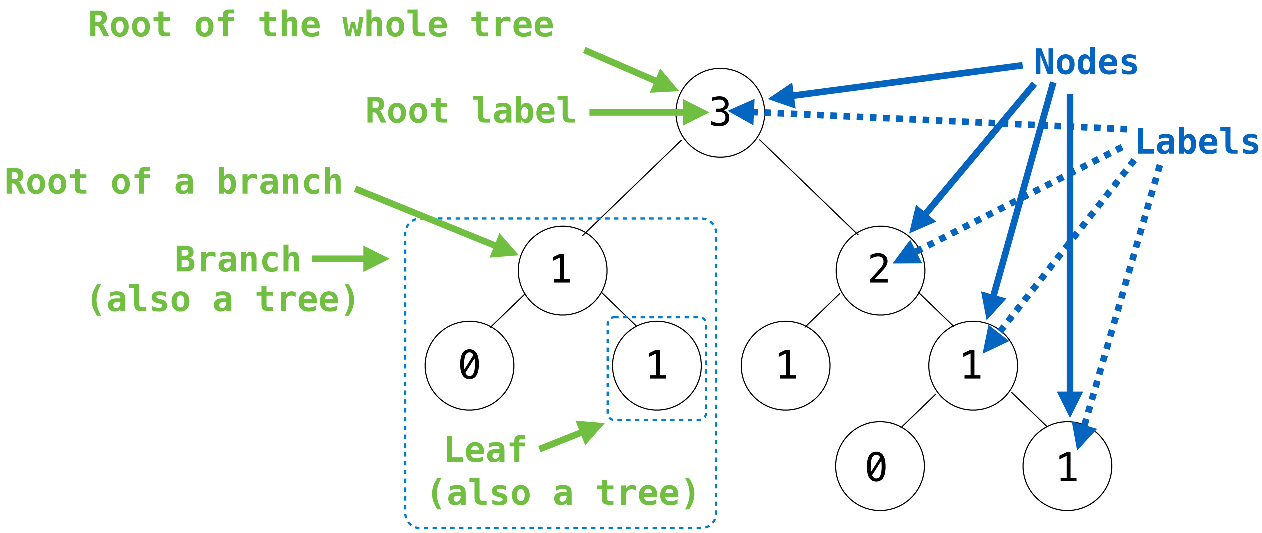

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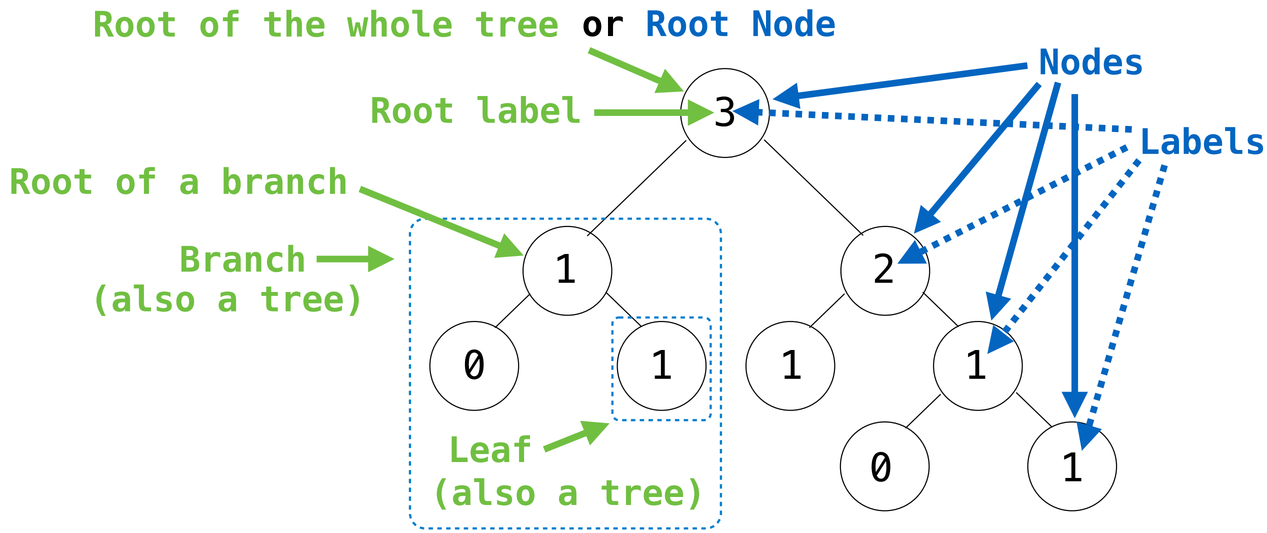

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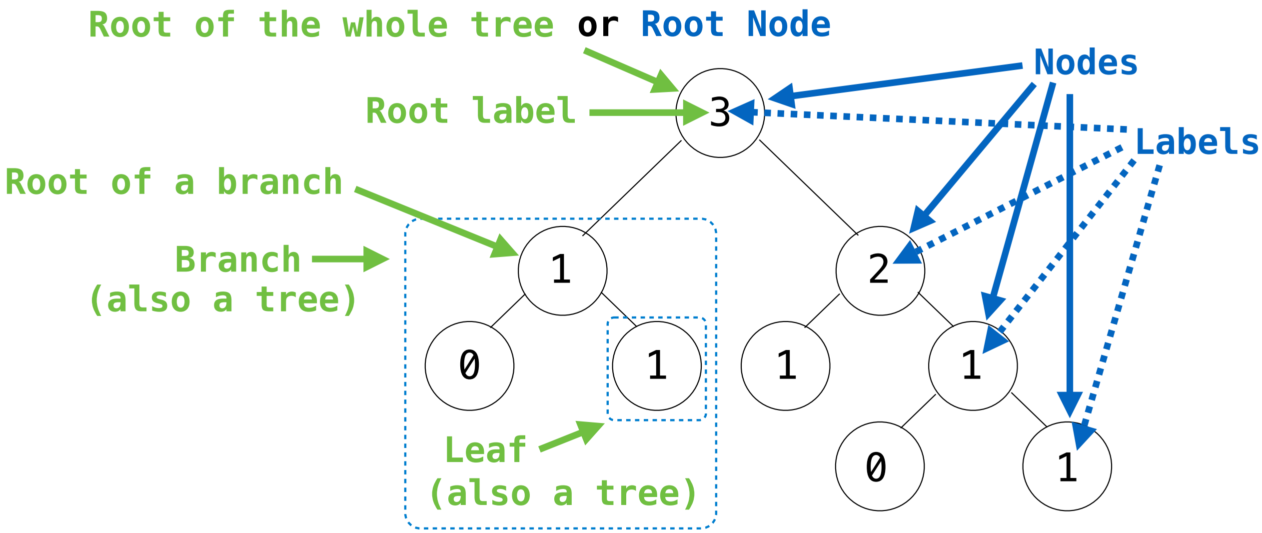

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People often refer to labels by their locations: "each parent is the sum of its children"

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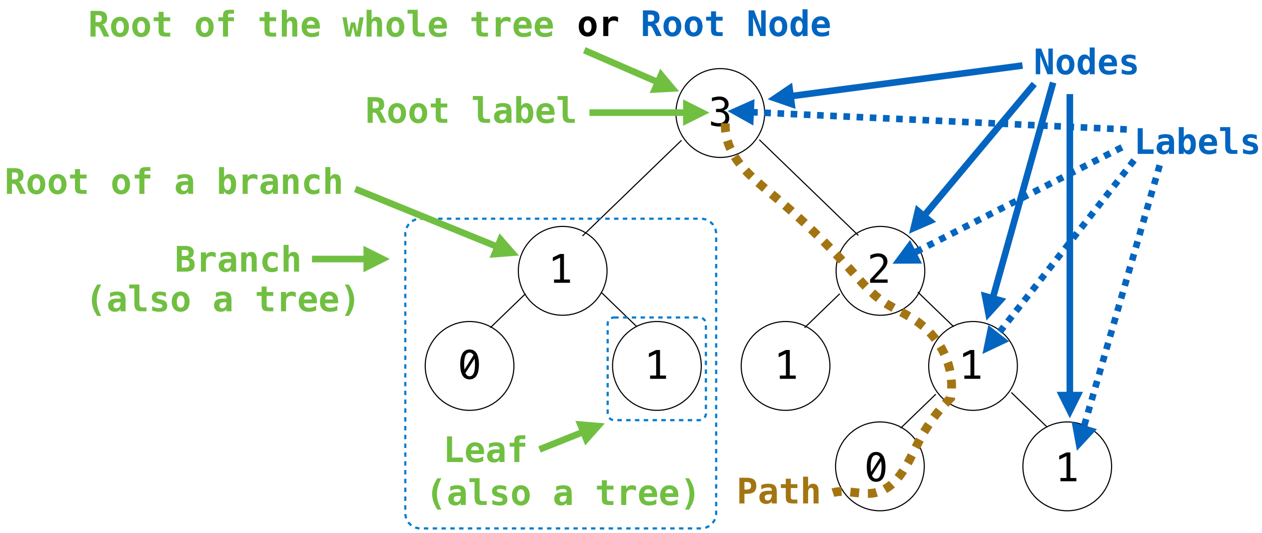

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Implementing the Tree Abstraction

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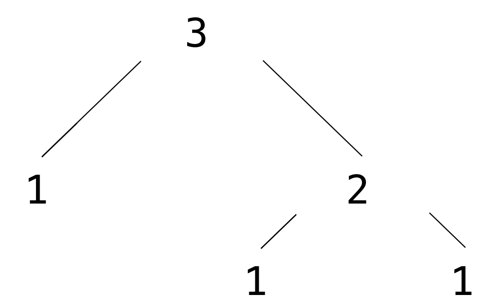

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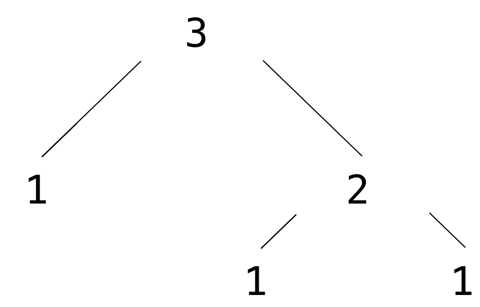


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>>> tree(3, [tree(1),
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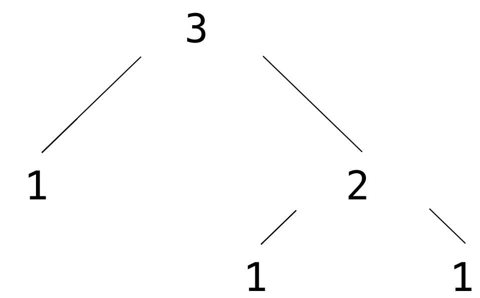


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def tree(label, branches=[]):
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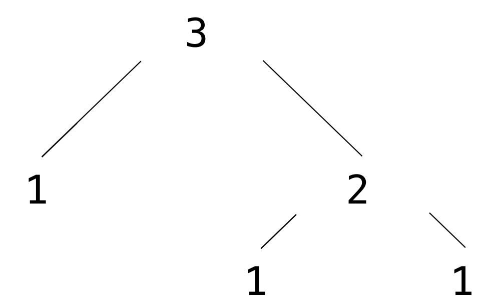


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def tree(label, branches=[]):
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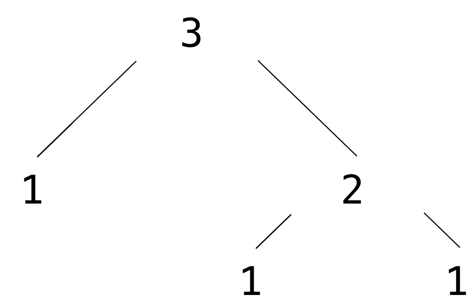


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def label(tree):
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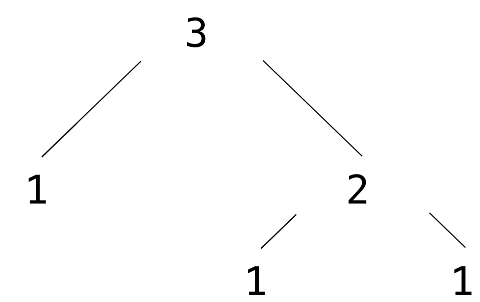


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def label(tree):
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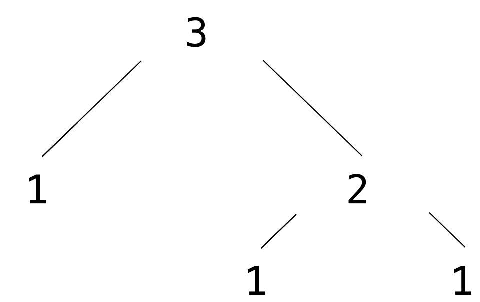


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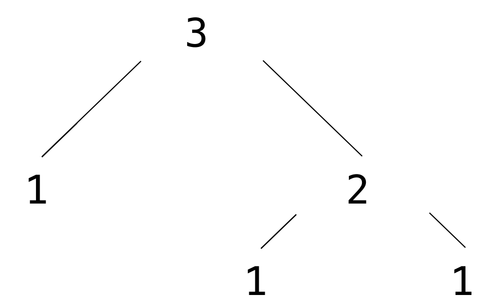


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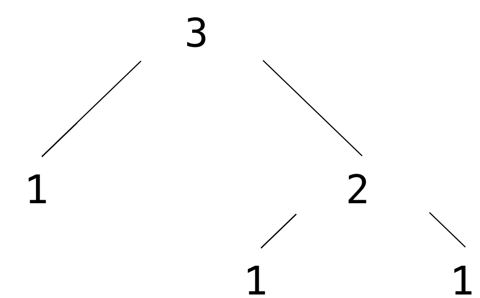


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def tree(label, branches=[]):
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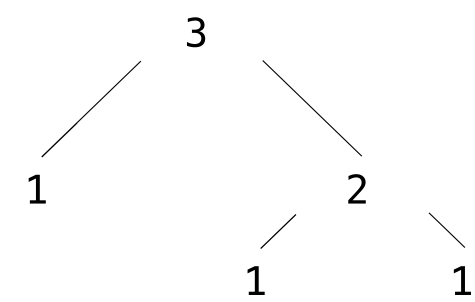


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Creates a list from a sequence of branches

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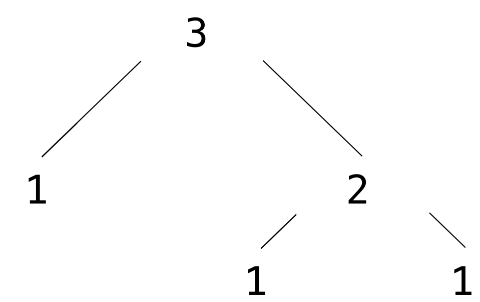


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Verifies the tree definition

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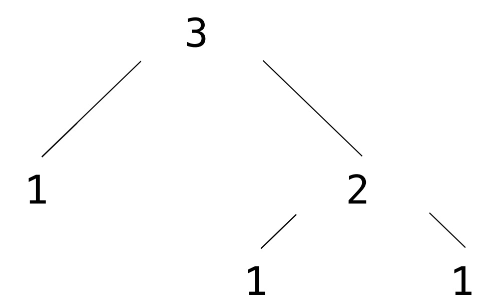


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def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
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    for branch in branches(tree):
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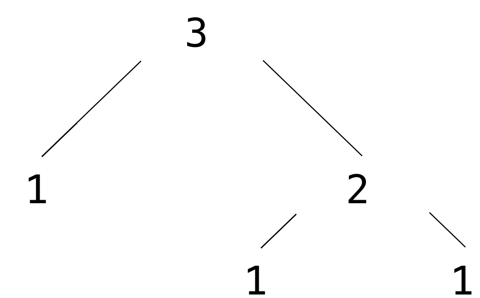


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Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

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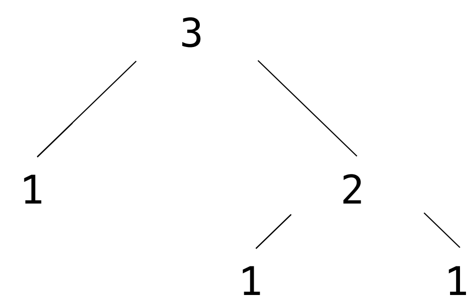


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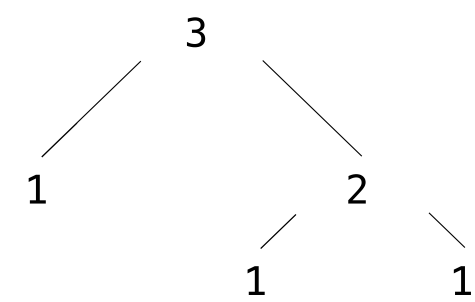


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def is_leaf(tree):
    return not branches(tree)
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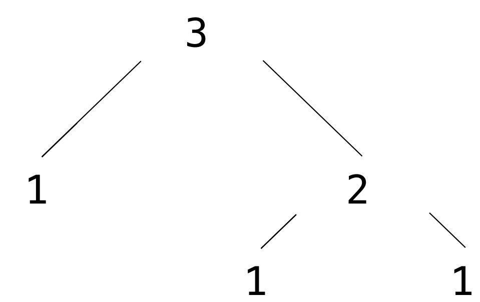


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...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]

def is_leaf(tree):
    return not branches(tree)
```

(Demo)

Tree Processing

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(Demo)

Tree Processing Uses Recursion

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(Demo)

Discussion Question

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def leaves(tree):
    """Return a list containing the leaf labels of tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
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Implement `leaves`, which returns a list of the leaf labels of a tree

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Example: Printing Trees

(Demo)

Example: Summing Paths

(Demo)

Example: Counting Paths

Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3)
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    """
    if __________________:

        found = _________________

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        _________________________

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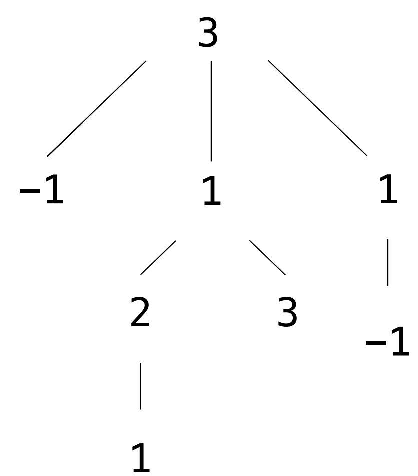

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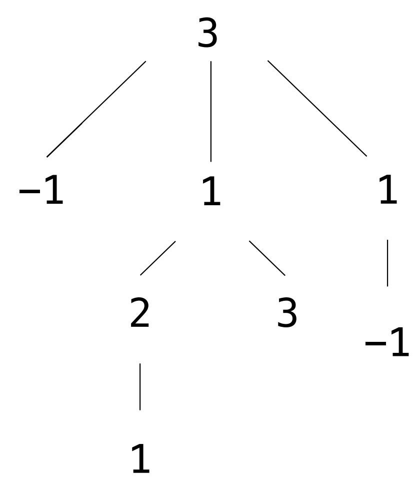

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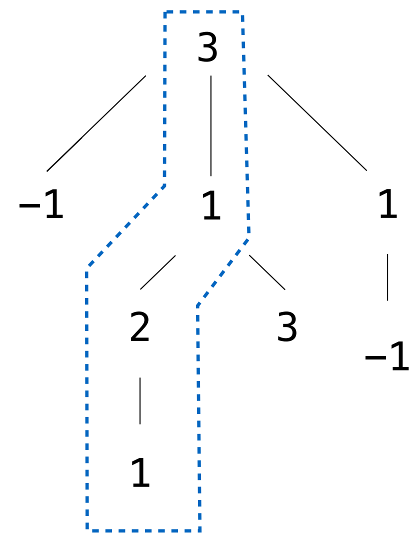

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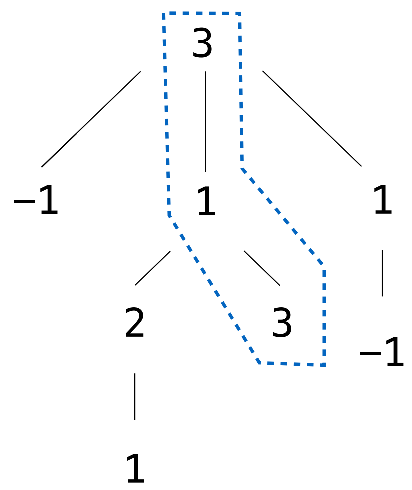

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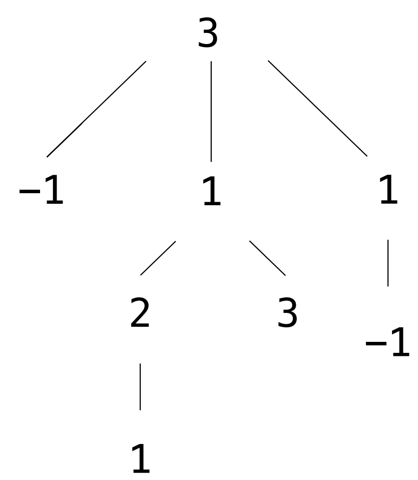

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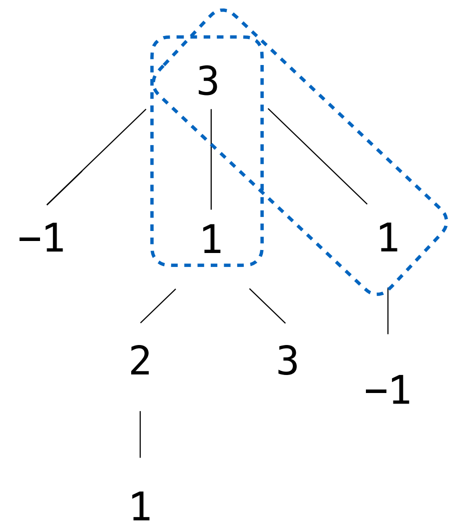

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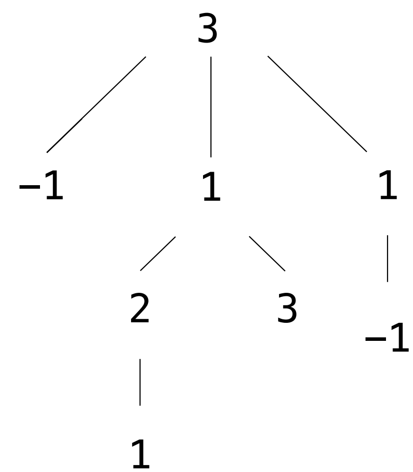

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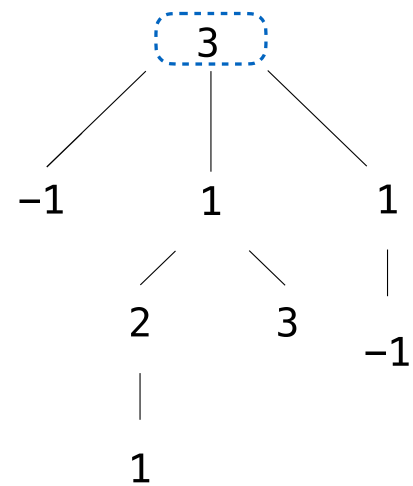

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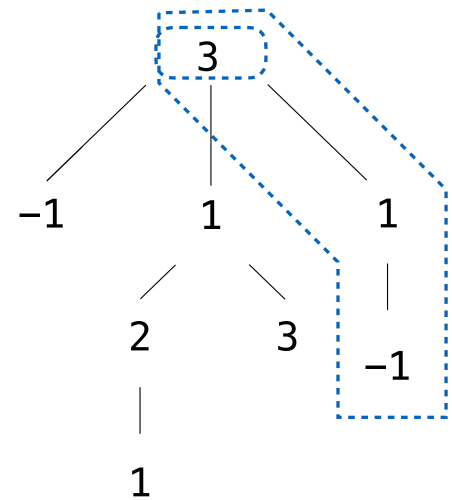

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    else:

        ________________________

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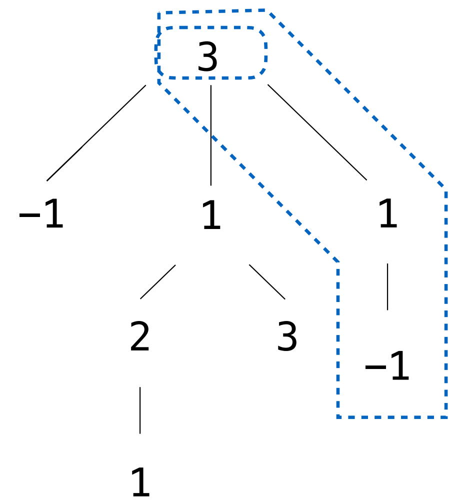

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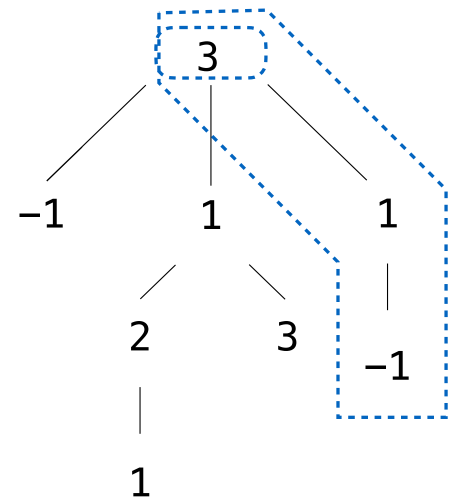

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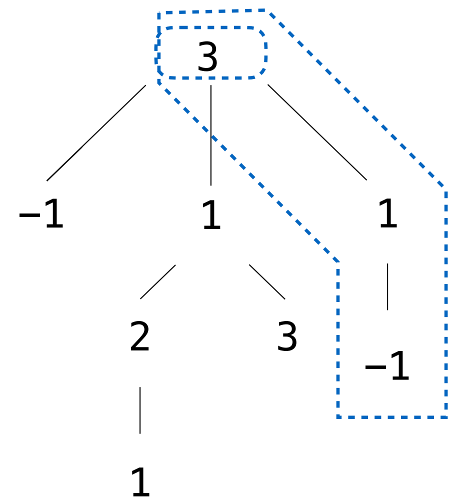

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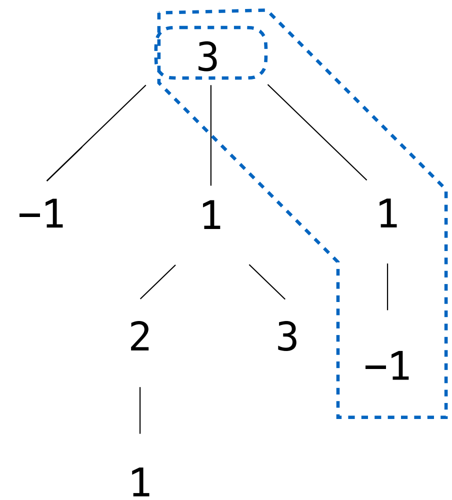
    if label(t) == total:
        found = ______1______
    else:
        found = 0_______________
    return found + ___sum___([_________________________________ for b in branches(t)])
```



Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:

        found = 1

    else:

        found = 0

    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
```