Mutable Functions
Announcements
Mutable Functions
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

>>> withdraw(25)
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
>>> withdraw(25)
75
```
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
>>> withdraw(25)
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Argument: amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
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Argument: amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
Argument: amount to withdraw
Return value: remaining balance

>>> withdraw(25)
75

>>> withdraw(25)
50
```
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
75

>>> withdraw(25)
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Argument: amount to withdraw

Second withdrawal of the same amount
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

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Different return value!

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Argument: amount to withdraw

Second withdrawal of the same amount
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Different return value!

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Argument: amount to withdraw

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'
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Return value: remaining balance

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Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

!!!
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'Insufficient funds'

!!!
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A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

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>>> withdraw(25)
75
```

Argument: amount to withdraw

```python
>>> withdraw(25)
50
```

Second withdrawal of the same amount

```python
>>> withdraw(60)
'Insufficient funds'
```

Different return value!

```python
>>> withdraw(15)
35
```

Where's this balance stored?
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```
>>> withdraw = make_withdraw(100)
```

Return value: remaining balance

```
>>> withdraw(25)
75
```

Argument: amount to withdraw

Second withdrawal of the same amount

```
>>> withdraw(25)
50
```

Where's this balance stored?

```
>>> withdraw(60)
'Insufficient funds'
```

```
>>> withdraw(15)
35
```

Different return value!
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
>>> withdraw = make_withdraw(100)
>>> withdraw(25)
75
>>> withdraw(25)
50
>>> withdraw(60)
'Insufficient funds'
>>> withdraw(15)
35
```

Within the parent frame of the function!

Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

Where's this balance stored?
Persistent Local State Using Environments

```python
def make_withdraw(balance):
    def withdraw(amount):
        balance = balance - amount
        return balance
    return withdraw

withdraw = make_withdraw(50)
withdraw(25)  # Returns 25
withdraw(25)  # Returns 50
```
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function

Every call decreases the same balance by (a possibly different) amount
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.

All calls to the same function have the same parent.

Every call decreases the same balance by (a possibly different) amount.
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
    return 100 * difference / x

diff = percent_difference(40, 50)
```

Global frame

```
percent_difference

f1: percent_difference [parent=Global]
```

<p>| | |</p>
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difference 10
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
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Assignment binds name(s) to value(s) in the first frame of the current environment.
Reminder: Local Assignment

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**Global frame**

```python
percent_difference
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**f1: percent_difference [parent=Global]**

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**difference**

= 10
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
    return 100 * difference / x

diff = percent_difference(40, 50)
```

**Execution rule for assignment statements:**
- Assignment binds name(s) to value(s) in the first frame of the current environment.
Reminder: Local Assignment

**Execution rule for assignment statements:**

1. Evaluate all expressions right of =, from left to right

2. Bind the names on the left to the resulting values in the **current frame**
Non-Local Assignment & Persistent Local State
Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
```

def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
def make_withdraw(balance):
    
    """Return a withdraw function with a starting balance."""

    def withdraw(amount):


def make_withdraw(balance):
    
    """Return a withdraw function with a starting balance."""

    def withdraw(amount):
        nonlocal balance
Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""

def withdraw(amount):
    nonlocal balance
    if amount > balance:
```

def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""

def withdraw(amount):
    nonlocal balance
    if amount > balance:
        return 'Insufficient funds'
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def withdraw(amount):
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    if amount > balance:
        return 'Insufficient funds'
    balance = balance - amount
Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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```

Non-Local Assignment & Persistent Local State

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def withdraw(amount):
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return withdraw
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Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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def withdraw(amount):
    nonlocal balance
    if amount > balance:
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    balance = balance - amount
    return balance

return withdraw
```

- Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned.
- Re-bind balance in the first non-local frame in which it was bound previously.
Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""

def withdraw(amount):
    nonlocal balance
    if amount > balance:
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    return balance

return withdraw
```

(Demo)
Non-Local Assignment
The Effect of Nonlocal Statements

nonlocal <name>
The Effect of Nonlocal Statements

nonlocal <name>

**Effect:** Future assignments to that name change its pre-existing binding in the first non-local frame of the current environment in which that name is bound.
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The Effect of Nonlocal Statements

nonlocal <name>, <name>, ...

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Python Docs: an "enclosing scope"
The Effect of Nonlocal Statements

nonlocal <name>, <name>, ...

**Effect:** Future assignments to that name change its pre-existing binding in the first non-local frame of the current environment in which that name is bound.

From the Python 3 language reference:
The Effect of Nonlocal Statements

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nonlocal <name>, <name>, ...
```

**Effect:** Future assignments to that name change its pre-existing binding in the first non-local frame of the current environment in which that name is bound.

From the Python 3 language reference:

Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.
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Names listed in a nonlocal statement must not collide with pre-existing bindings in the local scope.
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nonlocal <name>, <name>, ...
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https://docs.python.org/3/reference/simple_stmts.html#the-nonlocal-statement
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http://www.python.org/dev/peps/pep-3104/
The Many Meanings of Assignment Statements

\[ x = 2 \]
The Many Meanings of Assignment Statements

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### The Many Meanings of Assignment Statements

\[ x = 2 \]

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•"x" also bound locally | SyntaxError: name 'x' is parameter and nonlocal |
Python Particulars
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Python pre-computes which frame contains each name before executing the body of a function.
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Python pre-computes which frame contains each name before executing the body of a function. Within the body of a function, all instances of a name must refer to the same frame.
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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
```
Python Particulars

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        if amount > balance:
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wd = make_withdraw(20)
wd(5)
```

*UnboundLocalError: local variable 'balance' referenced before assignment*
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

```python
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
draw(25)
```
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

```python
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
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        b[0] = b[0] - amount
        return b[0]
    return withdraw
withdraw = make_withdraw_list(100)
withdraw(25)
```

Name bound outside of withdraw def
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

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withdraw = make_withdraw_list(100)
withdraw(25)
```
Mutable values can be changed *without* a nonlocal statement.

Name-value binding cannot change because there is no nonlocal statement.

Element assignment changes a list.
Mutable Values & Persistent Local State

Mutable values can be changed without a nonlocal statement.

```python
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
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Multiple Mutable Functions

(Demo)
Referential Transparency, Lost
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• Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.
Referential Transparency, Lost

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```
mul(add(2, mul(4, 6)), add(3, 5))
```
Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

\[
\text{mul(add(2, mul(4, 6)), add(3, 5))}
\]

\[
\text{mul(add(2, 24), add(3, 5))}
\]
Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

\[
\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
\]

\[
\text{mul}(\text{add}(2, 24), \text{add}(3, 5))
\]

\[
\text{mul}(26, \text{add}(3, 5))
\]
• Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

```python
mul(add(2, mul(4, 6)), add(3, 5))

mul(add(2, 24), add(3, 5))

mul(26, add(3, 5))
```

• Mutation operations violate the condition of referential transparency because they do more than just return a value; **they change the environment.**
Expresssions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

\[
\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
\]

\[
\text{mul}(\text{add}(2, 24), \text{add}(3, 5))
\]

\[
\text{mul}(26, \text{add}(3, 5))
\]

Mutation operations violate the condition of referential transparency because they do more than just return a value; they change the environment.
Referential Transparency, Lost

• Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

\[
\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
\]

\[
\text{mul}(\text{add}(2, 24), \text{add}(3, 5))
\]

\[
\text{mul}(26, \text{add}(3, 5))
\]

• Mutation operations violate the condition of referential transparency because they do more than just return a value; they **change the environment**.
Review Problem