Linked Lists
Class outline:

- Linked lists
- The Link class
- Processing linked lists
- Mutating linked lists
- Performance showdown
- Recursive objects
Linked lists
Why do we need a new list?

Python lists are implemented as a "dynamic array", which isn't optimal for all use cases.

✿ Inserting an element is slow, especially near front of list:

<table>
<thead>
<tr>
<th>&quot;A&quot;</th>
<th>&quot;B&quot;</th>
<th>&quot;C&quot;</th>
<th>&quot;D&quot;</th>
<th>&quot;E&quot;</th>
<th>&quot;F&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3300</td>
<td>3301</td>
<td>3302</td>
<td>3303</td>
<td>3304</td>
<td>3305</td>
</tr>
</tbody>
</table>

What should we insert?

value: Z @ index: 3 ✦ Insert
Why do we need a new list?

Python lists are implemented as a "dynamic array", which isn't optimal for all use cases.

 Decorating an element is slow, especially near front of list:

<p>| | | | | | |</p>
<table>
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<tr>
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What should we insert?

value: Z @ index: 3 → Insert

 Decorating too many elements can require re-creating the entire list in memory, if it exceeds the pre-allocated memory.
Linked lists

A linked list is a chain of objects where each object holds a **value** and a **reference to the next link**. The list ends when the final reference is empty.

What should we insert?

value: \[
\text{Z}\]
@ index: \[
5\]

Insert
Linked lists

A linked list is a chain of objects where each object holds a **value** and a **reference to the next link**. The list ends when the final reference is empty.

What should we insert?

value: **Z** @ index: 5 ➲ Insert

Linked lists require more space but provide faster insertion.
The Link class
A Link class

class Link:
    empty = ()

def __init__(self, first, rest=empty):
    self.first = first
    self.rest = rest

How would we use that?
A Link class

class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

How would we use that?

ll = Link("A", Link("B", Link("C")))

Try in PythonTutor
A fancier LinkedList

class Link:
    """A linked list.""
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '*
            self = self.rest
        return string + str(self.first) + '>'

It's built-in to code.cs61a.org and you can draw() any Link.
Creating linked lists
Creating a range

Similar to \[[x \text{ for } x \text{ in } \text{range}(3, 6)]\]

def range_link(start, end):
    """Return a Link containing consecutive integers from START to END, not including END.
    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """

Try in PythonTutor
Creating a range

Similar to \[x\text{ for }x\text{ in }\text{range}(3,\ 6)\]

```python
def range_link(start, end):
    """Return a Link containing consecutive integers from START to END, not including END."
    
    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """
    if start >= end:
        return Link.empty
    return Link(start, range_link(start + 1, end))
```

Try in PythonTutor
Exercise: Mapping a linked list

Similar to \( \{ f(x) \text{ for } x \text{ in } \text{lst} \} \)

```python
def map_link(f, ll):
    """Return a Link that contains f(x) for each x in Link LL."
    >>> square = lambda x: x * x
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """
```

Try in PythonTutor
Exercise: Mapping a linked list (Solution)

Similar to \[ f(x) \text{ for } x \in \text{lst} \]

```python
def map_link(f, ll):
    """Return a Link that contains f(x) for each x in Link LL.
    >>> square = lambda x: x * x
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """
    if ll is Link.empty:
        return Link.empty
    return Link(f(ll.first), map_link(f, ll.rest))
```

Try in PythonTutor
Exercise: Filtering a linked list

Similar to \[
\{x \text{ for } x \text{ in } lst \text{ if } f(x)\}\]

```python
def filter_link(f, ll):
    """Return a Link that contains only the elements x of Link LL for which f(x) is a true value."
    >>> is_odd = lambda x: x % 2 == 1
    >>> filter_link(is_odd, range_link(3, 6))
    Link(3, Link(5))
    """
```

Try in PythonTutor
Exercise: Filtering a linked list (Solution)

Similar to \[x \text{ for } x \text{ in } \text{lst if } f(x)\]

```python
def filter_link(f, ll):
    """Return a Link that contains only the elements \( x \) of Link \( LL \)
    for which \( f(x) \) is a true value.
    >>> is_odd = lambda x: x % 2 == 1
    >>> filter_link(is_odd, range_link(3, 6))
    Link(3, Link(5))
    """
    if ll is Link.empty:
        return Link.empty
    elif f(ll.first):
        return Link(ll.first, filter_link(f, ll.rest))
    return filter_link(f, ll.rest)
```

Try in PythonTutor
Mutating linked lists
Linked lists can change

Attribute assignments can change \texttt{first} and \texttt{rest} attributes of a \texttt{Link}.

\hspace{1cm} \texttt{s} = \texttt{Link("A", \texttt{Link("B", \texttt{Link("C")})})
Linked lists can change

Attribute assignments can change first and rest attributes of a Link.

```python
s = Link("A", Link("B", Link("C")))

s.first = "Hi"
s.rest.first = "Hola"
s.rest.rest.first = "Oi"
```

Try in PythonTutor
Beware infinite lists

The rest of a linked list can contain the linked list as a sub-list.

```plaintext
s = Link("A", Link("B", Link("C")))
t = s.rest
t.rest = s
```

```plaintext
s.first
```

```plaintext
s.rest.rest.rest.rest.rest.first
```
Exercise: Adding to front of linked list

```python
def insert_front(linked_list, new_val):
    """Inserts NEW_VAL in front of LINKED_LIST, returning new linked list."""

    >>> ll = Link(1, Link(3, Link(5)))
    >>> insert_front(ll, 0)
    Link(0, Link(1, Link(3, Link(5)))))
    """
```
Exercise: Adding to front of linked list (Solution)

```python
def insert_front(linked_list, new_val):
    """Inserts NEW_VAL in front of LINKED_LIST, returning new linked list.
    """
    return Link(new_val, linked_list)
```

```python
>>> ll = Link(1, Link(3, Link(5)))
>>> insert_front(ll, 0)
Link(0, Link(1, Link(3, Link(5))))
```

Exercise: Adding to an ordered linked list

```
def add(ordered_list, new_val):
    """Add NEW_VAL to ORDERED_LIST, returning modified ORDERED_LIST."
    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))))
    """
    if new_val < ordered_list.first:
        
    elif new_val > ordered_list.first and ordered_list.rest is Link.empty:
        
    elif new_val > ordered_list.first:
        
    return ordered_list
```
Exercise: Adding to an ordered linked list (Solution)

```python
def add(ordered_list, new_val):
    """Add NEW_VAL to ORDERED_LIST, returning modified ORDERED_LIST."
    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6)))))
    """
    if new_val < ordered_list.first:
        original_first = ordered_list.first
        ordered_list.first = new_val
        ordered_list.rest = Link(original_first, ordered_list.rest)
    elif new_val > ordered_list.first and ordered_list.rest is Link.empty:
        ordered_list.rest = Link(new_val)
    elif new_val > ordered_list.first:
        add(ordered_list.rest, new_val)
    return ordered_list
```

Insert value: 0 @ index: 0 Insert
Showdown: Python list vs. Link

The challenge:

- Store all the half-a-million words in "War and Peace"
- Insert a word at the beginning.

<table>
<thead>
<tr>
<th>Version</th>
<th>10,000 runs</th>
<th>100,000 runs</th>
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<tbody>
<tr>
<td>Python list</td>
<td></td>
<td></td>
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<tr>
<td>Link</td>
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Try it yourself on your local machine (Legit Python!): warandpeace.py
Showdown: Python list vs. Link

The challenge:

- Store all the half-a-million words in "War and Peace"
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Showdown: Python list vs. Link

The challenge:

- Store all the half-a-million words in "War and Peace"
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<tbody>
<tr>
<td>Python list</td>
<td>2.6 seconds</td>
<td>37 seconds</td>
</tr>
<tr>
<td>Link</td>
<td>0.01 seconds</td>
<td>0.1</td>
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Try it yourself on your local machine (Legit Python!): warandpeace.py
Recursive objects
Recursive objects

Why are Tree and Link considered recursive objects?
Recursive objects

Why are `Tree` and `Link` considered recursive objects?

Each type of object contains references to the same type of object.

- An instance of `Tree` can contain additional instances of `Tree`, in the `branches` variable.
- An instance of `Link` can contain an additional instance of `Link`, in the `rest` variable.

Both classes lend themselves to recursive algorithms. Generally:

- For `Tree`: The base case is when `is_leaf()` is true; the recursive call is on the `branches`.
- For `Link`: The base case is when the rest is `empty`; the recursive call is on the `rest`.