Linked Lists
Class outline:

- Linked lists
- The Link class
- Processing linked lists
- Mutating linked lists
- Performance showdown
- Recursive objects
Linked lists
Why do we need a new list?

Python lists are implemented as a "dynamic array", which isn't optimal for all use cases.

😊 Inserting an element is slow, especially near front of list:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>&quot;A&quot;</td>
<td>&quot;B&quot;</td>
<td>&quot;C&quot;</td>
<td>&quot;D&quot;</td>
<td>&quot;E&quot;</td>
<td>&quot;F&quot;</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3300</td>
<td>3301</td>
<td>3302</td>
<td>3303</td>
<td>3304</td>
<td>3305</td>
</tr>
</tbody>
</table>

What should we insert?

value: [Z] @ index: [3]  Insert
Why do we need a new list?

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-inserting an element is slow, especially near front of list:

<table>
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<tr>
<td>0</td>
<td>1</td>
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What should we insert?

-value: \[Z\] @ index: 3

-inserting too many elements can require re-creating the entire list in memory, if it exceeds the pre-allocated memory.
Linked lists

A linked list is a chain of objects where each object holds a value and a reference to the next link. The list ends when the final reference is empty.

What should we insert?

value: Z @ index: 5 Insert
Linked lists

A linked list is a chain of objects where each object holds a **value** and a **reference to the next link**. The list ends when the final reference is empty.

What should we insert?

**value:** Z @ **index:** 5  
**Insert**

Linked lists require more space but provide faster insertion.
The Link class
A Link class

class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

How would we use that?
A Link class

class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

How would we use that?

ll = Link("A", Link("B", Link("C")))

Try in PythonTutor
A fancier LinkedList

class Link:
    """A linked list."""
    empty = ()

def __init__(self, first, rest=empty):
    assert rest is Link.empty or isinstance(rest, Link)
    self.first = first
    self.rest = rest

def __repr__(self):
    if self.rest:
       rest_repr = ', ' + repr(self.rest)
    else:
       rest_repr = ''
    return 'Link(' + repr(self.first) + rest_repr + ')

def __str__(self):
    string = '<'
    while self.rest is not Link.empty:
        string += str(self.first) + ' '  
        self = self.rest
    return string + str(self.first) + '>'

It's built-in to code.cs61a.org and you can draw() any Link.
Creating linked lists
def range_link(start, end):
    """Return a Link containing consecutive integers
    from START to END, not including END.
    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """
Creating a range

Similar to \[x \text{ for } x \text{ in } \text{range}(3, 6)\]

def range_link(start, end):
    """Return a Link containing consecutive integers from START to END, not including END."
    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """
    if start >= end:
        return Link.empty
    return Link(start, range_link(start + 1, end))

Try in PythonTutor
Exercise: Mapping a linked list

Similar to  

```python
def map_link(f, ll):
    """Return a Link that contains f(x) for each x in Link LL.
    >>> square = lambda x: x * x
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """
```

Try in PythonTutor
Exercise: Mapping a linked list (Solution)

Similar to  \[ f(x) \text{ for } x \text{ in } lst \]

```python
def map_link(f, ll):
    '''Return a Link that contains f(x) for each x in Link LL.'''
    >>> square = lambda x: x * x
    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    '''
    if ll is Link.empty:
        return Link.empty
    return Link(f(ll.first), map_link(f, ll.rest))
```

Try in PythonTutor
Exercise: Filtering a linked list

Similar to \[x \text{ for } x \text{ in } \text{lst} \text{ if } f(x)\]

```python
def filter_link(f, ll):
    """Return a Link that contains only the elements x of Link LL
    for which f(x) is a true value.
    >>> is_odd = lambda x: x % 2 == 1
    >>> filter_link(is_odd, range_link(3, 6))
    Link(3, Link(5))
    """
```

Try in PythonTutor
Exercise: Filtering a linked list (Solution)

Similar to `[x for x in lst if f(x)]`

def filter_link(f, ll):
    """Return a Link that contains only the elements x of Link LL
    for which f(x) is a true value.
    >>> is_odd = lambda x: x % 2 == 1
    >>> filter_link(is_odd, range_link(3, 6))
    Link(3, Link(5))
    """
    if ll is Link.empty:
        return Link.empty
    elif f(ll.first):
        return Link(ll.first, filter_link(f, ll.rest))
    return filter_link(f, ll.rest)

Try in PythonTutor
Mutating linked lists
Linked lists can change

Attribute assignments can change first and rest attributes of a Link.

\[
s = \text{Link}("A", \text{Link}("B", \text{Link}("C")))
\]
Linked lists can change

Attribute assignments can change first and rest attributes of a Link.

```
s = Link("A", Link("B", Link("C")))
```

```
s.first = "Hi"
s.rest.first = "Hola"
s.rest.rest.first = "Oi"
```

Try in PythonTutor
Beware infinite lists

The rest of a linked list can contain the linked list as a sub-list.

```scheme
s = Link("A", Link("B", Link("C")))
t = s.rest
t.rest = s
```

```scheme
s.first
```

```scheme
s.rest.rest.rest.rest.rest.first
```
Exercise: Adding to front of linked list

```python
def insert_front(linked_list, new_val):
    """Inserts NEW_VAL in front of LINKED_LIST, returning new linked list."""
    
    >>> ll = Link(1, Link(3, Link(5)))
    >>> insert_front(ll, 0)
    Link(0, Link(1, Link(3, Link(5)))))"""
```
Exercise: Adding to front of linked list (Solution)

```python
def insert_front(linked_list, new_val):
    """Inserts NEW_VAL in front of LINKED_LIST, returning new linked list."

    >>> ll = Link(1, Link(3, Link(5)))
    >>> insert_front(ll, 0)
    Link(0, Link(1, Link(3, Link(5))))
    """
    return Link(new_val, linked_list)
```

"""Insert"""
Exercise: Adding to an ordered linked list

```
def add(ordered_list, new_val):
    """Add NEW_VAL to ORDERED_LIST, returning modified ORDERED_LIST."
    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5)))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5)))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6)))))))
    """
    if new_val < ordered_list.first:
        ...  
    elif new_val > ordered_list.first and ordered_list.rest is Link.empty:
        ...
    elif new_val > ordered_list.first:
        ...
    return ordered_list

Insert value: 0  @ index: 0  Insert
```
Exercise: Adding to an ordered linked list (Solution)

```
def add(ordered_list, new_val):
    """Add NEW_VAL to ORDERED_LIST, returning modified ORDERED_LIST.
    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
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    Link(0, Link(1, Link(3, Link(5)))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6)))))))
    ""
    if new_val < ordered_list.first:
        original_first = ordered_list.first
        ordered_list.first = new_val
        ordered_list.rest = Link(original_first, ordered_list.rest)
    elif new_val > ordered_list.first and ordered_list.rest is Link.empty:
        ordered_list.rest = Link(new_val)
    elif new_val > ordered_list.first:
        add(ordered_list.rest, new_val)
    return ordered_list
```
Showdown: Python list vs. Link

The challenge:

- Store all the half-a-million words in "War and Peace"
- Insert a word at the beginning.

<table>
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<tr>
<th>Version</th>
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<th>100,000 runs</th>
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<tr>
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<td></td>
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Try it yourself on your local machine (Legit Python!): warandpeace.py
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Showdown: Python list vs. Link

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- Store all the half-a-million words in "War and Peace"
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<td>2.6 seconds</td>
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</tr>
<tr>
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<td>0.01 seconds</td>
<td>0.1</td>
</tr>
</tbody>
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Try it yourself on your local machine (Legit Python!): warandpeace.py
Recursive objects
Recursive objects

Why are Tree and Link considered recursive objects?
Recursive objects

Why are **Tree** and **Link** considered recursive objects?

Each type of object contains references to the same type of object.

- An instance of **Tree** can contain additional instances of **Tree**, in the **branches** variable.
- An instance of **Link** can contain an additional instance of **Link**, in the **rest** variable.

Both classes lend themselves to recursive algorithms. Generally:

- For **Tree**: The base case is when **is_leaf()** is true; the recursive call is on the **branches**.
- For **Link**: The base case is when the rest is **empty**; the recursive call is on the **rest**.