# Efficiency

# Announcements

# Tree Class

## Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

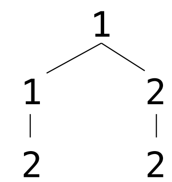
# Tree Practice

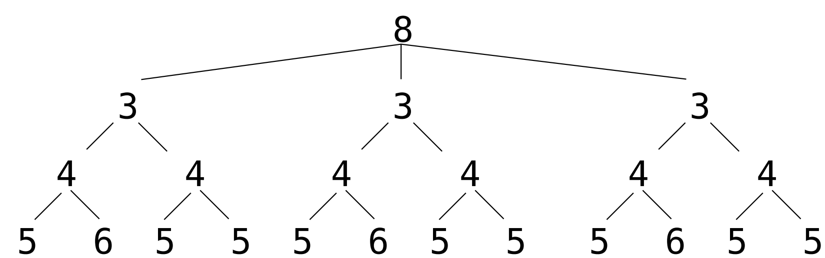
# Example: Count Twins

Implement twins, which takes a Tree t. It return the number of pairs of sibling nodes whose labels are equal.

```
def twins(t):
    """Count the pairs of sibling nodes with equal labels.

    >>> t1 = Tree(3, [Tree(4, [Tree(5), Tree(6)]), Tree(4, [Tree(5), Tree(5)])])
    >>> twins(t1)  # 4 and 5
    2
    >>> twins(Tree(1, [Tree(1, [Tree(2)]), Tree(2, [Tree(2)])]))
    0
    >>> twins(Tree(8, [t1, t1, t1]))  # 3 pairs of twins at the top, plus 2 in each branch
    9
    """
    count = 0
    n = len(t.branches)
    for i in range(n-1):
        for j in range(i+1, n):
            if t.branches[i].label == t.branches[j].label:
                count += 1
    return count + sum([twins(b) for b in t.branches])
```



## Spring 2023 Midterm 2 Question 4(b)

You have already implemented **exclude(t, x)**, which takes a Tree instance t and a value x. It returns a Tree containing the root node of t as well as each non-root node of t with a label not equal to x. The parent of a node in the result is its nearest ancestor node that is not excluded. The input t is not modified.

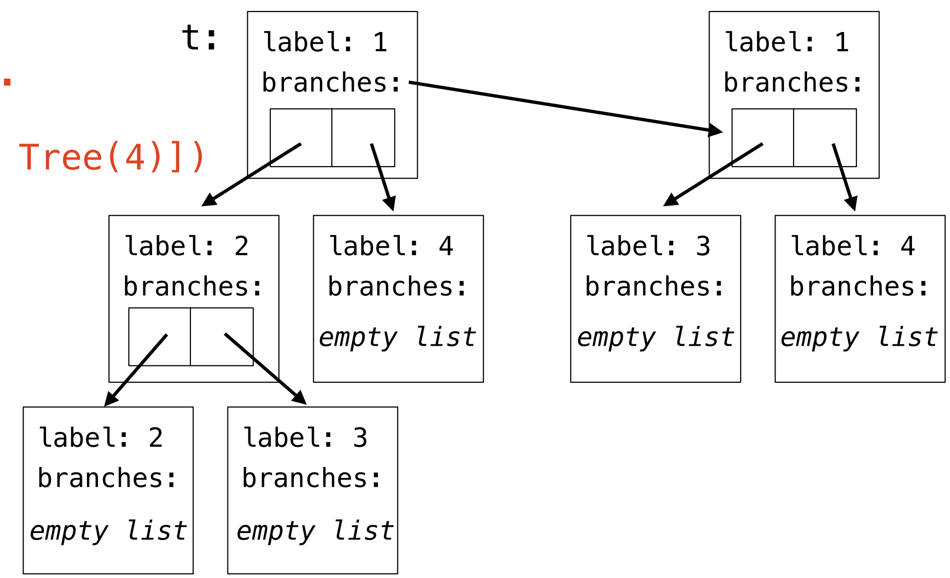
Implement **remove**, which takes a Tree instance t and a value x. It removes all non-root nodes from t that have a label equal to x, then returns t. The parent of a node in t is its nearest ancestor that is not removed.

```
def remove(t, x):
    """Remove all non-root nodes labeled x from t.

    >>> t = Tree(1, [Tree(2, [Tree(2), Tree(3)]), Tree(4)])
    >>> remove(t, 2)
    Tree(1, [Tree(3), Tree(4)])
    >>> remove(t, 3)
    Tree(1, [Tree(4)])
    """

    t.branches = exclude(t, x).branches
    ______________________________________

    return t
```

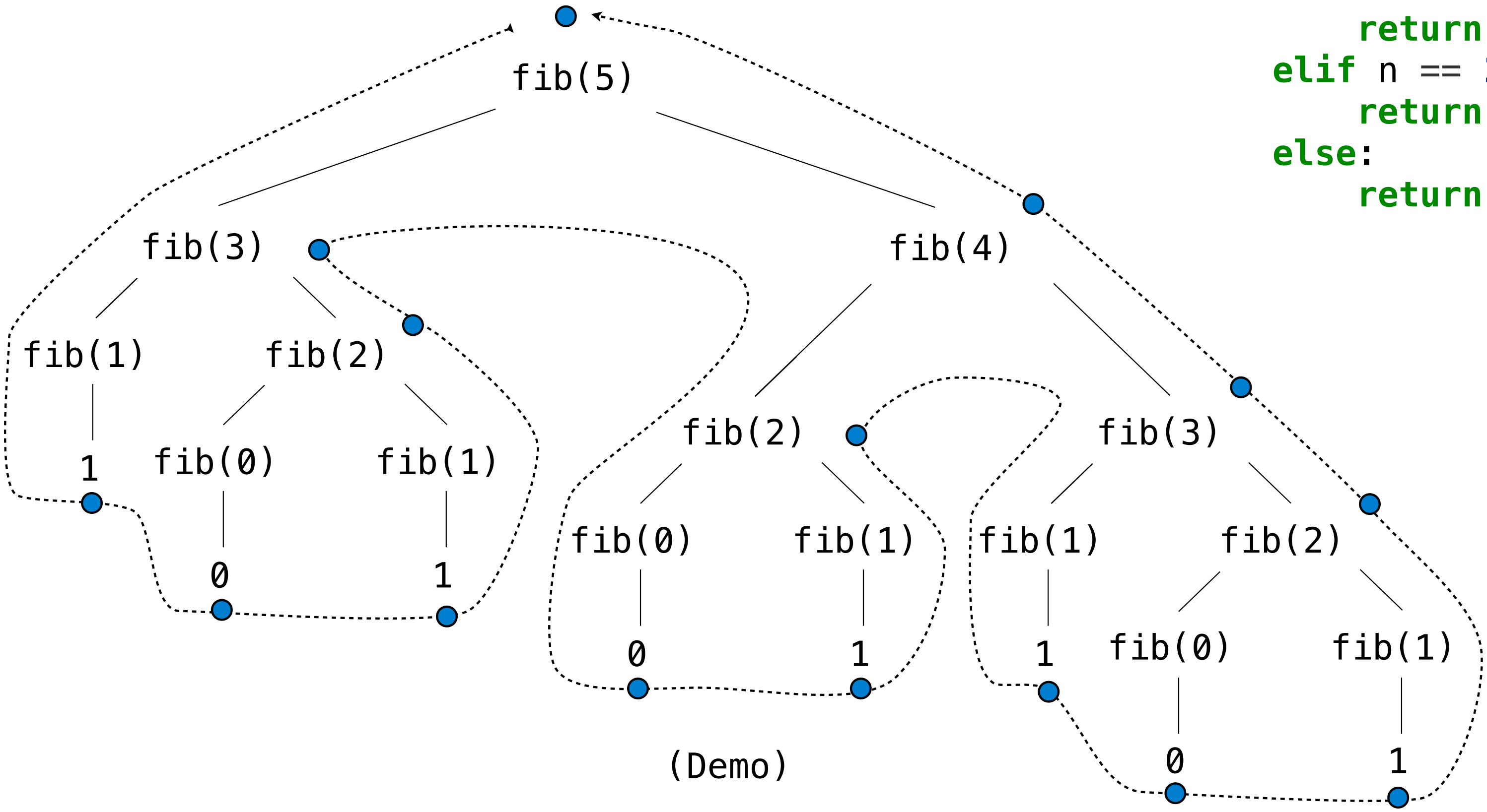


# Measuring Efficiency

# Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```




(Demo)

# Memoization

# Memoization

**Idea:** Remember the results that have been computed before

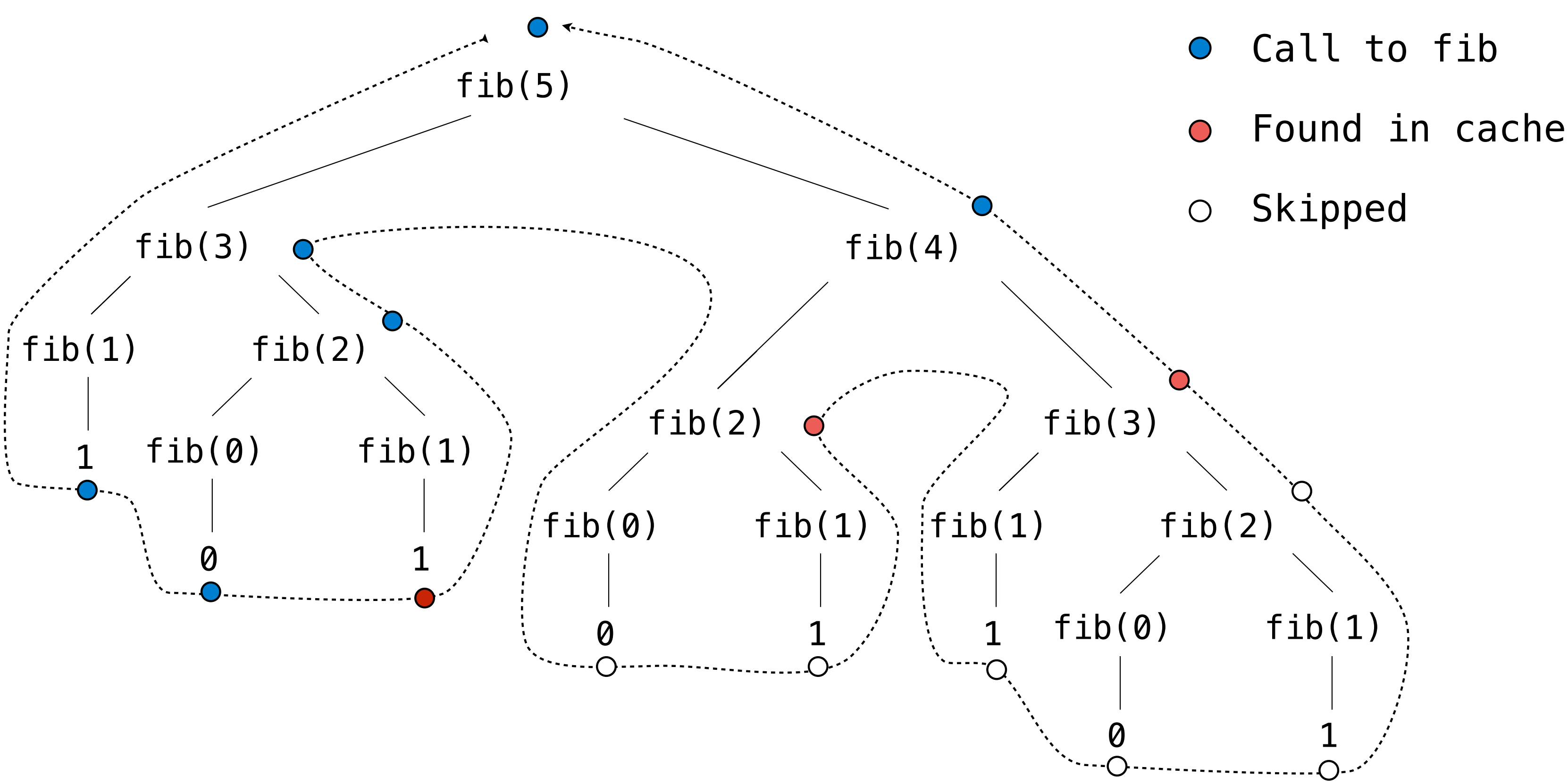
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

(Demo)

# Memoized Tree Recursion



# Orders of Growth

## Common Orders of Growth

**Exponential growth.** E.g., recursive `fib`

Incrementing *n* multiplies *time* by a constant

**Quadratic growth.**

Incrementing *n* increases *time* by *n* times a constant

**Linear growth.**

Incrementing *n* increases *time* by a constant

**Logarithmic growth.**

Doubling *n* only increments *time* by a constant

**Constant growth.** Increasing *n* doesn't affect time

## Spring 2023 Midterm 2 Question 3(a) Part (iii)

**Definition.** A *prefix sum* of a sequence of numbers is the sum of the first n elements for some positive length n.

(1 pt) What is the order of growth of the time to run prefix(s) in terms of the length of s? Assume append takes one step (constant time) for any arguments.

```
def prefix(s):
    "Return a list of all prefix sums of list s."
    t = 0
    result = []
    for x in s:
        t = t + x
        result.append(t)
    return result
```