Translate to Scheme

- Convert this Python program into Scheme:

  ```python
def count(predicate, L):
    if L is Link.empty:
      return 0
    elif predicate(L.first):
      return 1 + count(predicate, L.rest)
    else:
      return count(predicate, L.rest)
  ```

  Scheme version:

  ```scheme
(define (count predicate L)
  (cond ((null? L) 0)
        ((predicate (car L)) (+ 1 (count predicate (cdr L))))
        (else (count predicate (cdr L)))) ; in cond, else == #t
  )
  (count odd? '(1 12 13 19 4 6 9)) ==> 4
  (count odd? '()) ==> 0
)
```

- Is this tail-recursive?
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    else:
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```

**Scheme version:**
```
(define (count predicate L)
  (cond ((null? L) 0)
        ((predicate (car L)) (+ 1 (count predicate (cdr L))))
     ; Not a tail call
        (else (count predicate (cdr L))))
  ; in cond, else == #t
)
```

- Is this tail-recursive? **No**

Review of Iteration via Tail Recursion

- Earlier in the course, we saw that iterations are related to tail-recursions.

- Consider a general Python loop:

```python
def my_function(...):
    <variables> = <initial values>
    while <some condition>:
        <variables> = <new values>
        return <some value>
```

- Many programs can be put into this form, equivalent to

```python
def my_function(...):
    def looper(<variables>):
        if <some condition>:
            return looper(<new values>)
        else:
            return <some value>
    return looper(<initial values>)
```

- Converts directly into Scheme:

```scheme
(define (my_function ...) (define (looper <variables>)
  (if <some condition> (looper <new values>)
   <some value>)
  return <some value>
  return looper(<initial values>)
)
```

- Significance of this particular kind of recursion is that Scheme implementations (but not Python) must not fail regardless of the depth of the tail calls.

Review of Iteration via Tail Recursion (II)

- And this Python recursion:

```python
def my_function(...):
    def looper(<variables>):
        if <some condition>:
            return looper(<new values>)
        else:
            return <some value>
    return looper(<initial values>)
```

- Converts directly into Scheme:

```scheme
(define (my_function ...) (define (looper <variables>)
  (if <some condition> (looper <new values>)
   <some value>)
  (looper <initial values>))
```

- Significance of this particular kind of recursion is that Scheme implementations (but not Python) must not fail regardless of the depth of the tail calls.
Tail-Recursive Version of count

• First, the Python version:

```python
def count(predicate, L):
def count1(L, s):
    """Return S + # of items in L that satisfy PREDICATE."""
    return count1(L, 0)
```

• And now, Scheme:

```scheme
(define (count predicate L)
    ?
)
```
Tail-Recursive Version of count

- First, the Python version:
  ```python
def count(predicate, L):
def count1(L, s):
    """Return S + # of items in L that satisfy PREDICATE."""
    if L is Link.empty:
        return s
    elif predicate(L.first):
        return count1(L.rest, s + 1)
    else:
        return count1(L.rest, s)
return count1(L, 0)
```

- And now, Scheme:
  ```scheme
(define (count predicate L)
  (define (count1 L s)
    (cond ((null? L) s)
          ((predicate (car L)) (count1 (cdr L) (+ s 1)))
          (#t (count1 (cdr L) s))))
  (count1 L 0))
```

Another Higher-Order Function Example: Map

- We've seen `map` in Python, where it is built-in for iterables, and we can define it there for linked lists:
  ```python
def map(fn, L):
    if L is Link.empty:
        return Link.empty
    else:
        return Link(fn(L.first), map(fn, L.rest))
```

- What about in Scheme?
  ```scheme
(scm> (define (map fn L))
```

Another Higher-Order Function Example: Map

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    if L is Link.empty:
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        return Link(fn(L.first), map(fn, L.rest))
```

- What about in Scheme?
  ```scheme
(scm> (define (map fn L)
        (if (null? L)
            ))
```
Another Higher-Order Function Example: Map

- We've seen map in Python, where it is built-in for iterables, and we can define it there for linked lists:
  ```python
def map(fn, L):
    if L is Link.empty:
      return Link.empty
    else:
      return Link(fn(L.first), map(fn, L.rest))
  ```

- What about in Scheme?
  ```scheme
(define (map fn L)
  (if (null? L)(cons (fn (car L)) (map fn (cdr L)))))
)
```
Reverse

• Suppose we could write \(\text{reverse } L\) to get the reverse of a list:
  \[
  \text{scm> (reverse '(1 2 3))}
  (3 2 1)
  \]
• How could we use this to do map tail-recursively?
  \[
  \begin{align*}
  \text{(define (map fn L)} & \\
  \text{(define (loop list-so-far L)} & \\
  & \text{(if (null? L) list-so-far)} \\
  & \text{(loop (cons (fn (car L)) list-so-far) (cdr L))})
  \end{align*}
  \]
• So now we just have to get a tail-recursive reverse

Tail-Recursive Reverse

• Not really so difficult, once you think about how you realize that, for example,
  \[
  \text{scm> (define L '(1 2 3))}
  \text{scm> (reverse L)}
  (3 2 1)
  \text{scm> (cons (car (cdr (cdr L))) (cons (car (cdr L)) (cons (car L) '())))}
  (3 2 1)
  \]
• This might suggest the order in which the reversed list gets built, suggesting a program like this:
  \[
  \begin{align*}
  \text{(define (reverse L)} & \\
  \text{(define (reverse1 so-far L)} & \\
  & \text{(if (null? L) so-far)} \\
  & \text{(reverse1 (cons (car L) so-far) (cdr L))})
  \end{align*}
  \]
• This might suggest the order in which the reversed list gets built, suggesting a program like this:
  \[
  \begin{align*}
  \text{(define (reverse L)} & \\
  \text{(define (reverse1 so-far L)} & \\
  & \text{(if (null? L) so-far)} \\
  & \text{(reverse1 (cons (car L) so-far) (cdr L))})
  \end{align*}
  \]
Tail-Recursive Reverse

- Not really so difficult, once you think about how you realize that, for example,
  ```scheme
  scm> (define L '(1 2 3))
  scm> (reverse L)
  (3 2 1)
  scm> (cons (car (cdr (cdr L))) (cons (car (cdr L)) (cons (car L) '())))
  (3 2 1)
  - This might suggest the order in which the reversed list gets built, suggesting a program like this:
    ```scheme
    (define (reverse L)
        (define (reverse1 so-far L)
            (if (null? L) so-far
                (reverse1 (cons (car L) so-far) (cdr L))))
        (reverse1 '() L))
    ```

Trees

- How could we represent a tree in Scheme?
  ```scheme
  6
  3 5
  1 7
  1 8
  9
  ```
- Can use a representation similar to what we used in Python, such as
  `(6 (3 (1)) (5) (7 (8) (9)))`
- Abstracting into functions:
  ```scheme
  (define (tree label children) (cons label children))
  (define (label tr) (car tr))
  (define (children tr) (cdr tr))
  (define (is-leaf tr) (null? (cdr tr)))
  ```

Tree Recursions

- Assuming our labels are integers, how could we implement the label-doubling function from lecture 12 in Scheme?
  ```scheme
  (define (double tr)
      "Return a tree identical to TR, but with all labels doubled."
      (tree (* (label tr) 2) (map double (children tr))))
  ```

Tree Recursions

- Assuming our labels are integers, how could we implement the label-doubling function from lecture 12 in Scheme?
  ```scheme
  (define (double tr)
      "Return a tree identical to TR, but with all labels doubled."
      (tree (* (label tr) 2) (map double (children tr))))
  ```