Declarative vs. Imperative

- Up until now, we have dealt with what is called imperative programming.
- That is, our programs consist of commands that tell how to perform some computation.
- An alternative is declarative programming.
- Here, our programs describe the desired characteristics of the result, and leave it to the system to figure out how to get them.
- If feasible, it would seem that declarative programming is preferable, but that it is, of course, difficult to achieve.
- One current version is programming by example, in which one gives examples of desired results and tries to get the system to generalize this into an algorithm.
- Unsurprisingly, this hasn't quite made it to the main stream.
- There is a history of systems for making logical assertions about a problem domain and having the system figure out how to solve a problem in that domain.

Prolog and Predecessors

- Way back in 1959, AI researchers at CMU created GPS (General Problem Solver [A. Newell, J. C. Shaw, H. A. Simon])
  - Input defined objects and allowable operations on them, plus a description of the desired outcome.
  - Output consisted of a sequence of operations to bring the outcome about.
  - Only worked for small problems, unsurprisingly.
- Planner at MIT [C. Hewitt, 1969] was another programming language for theorem proving: one specified the desired goal assertion to be proven, and system would find rules to apply to demonstrate the assertion. Again, this didn't scale all that well.
- Planner was one inspiration for the development of the logic-programming language Prolog.
- Prolog was originally developed for AI applications (esp. natural-language processing) and is still in use for a variety of applications in AI.
- Figured in the Fifth Generation Computer Systems Project in Japan in the 1980s.

Prolog (Lisp Style)

- Let's interpret Scheme expressions as logical assertions.
- For example, (likes brian potstickers) might be such an assertion: likes is a predicate that relates brian and potstickers.
- We don't interpret the arguments of the predicate: as far as Scheme is concerned they are just uninterpreted data structures.
- We also allow one other type of expression: a symbol that starts with a question mark will indicate a logical variable.
- An assertion such as (likes brian ?X) asserts that there is some replacement for ?X that makes the assertion true.
Facts and Rules

• We will make queries in the form of assertions, possibly with logical variables.

• The system will look to see if the queries are true based on a database of facts (axioms or postulates) about the predicates.

• It will inform us of what replacements for logical variables make the assertion true.

• Each fact will have the form
  \[(\text{fact Conclusion Hypothesis}_1 \text{ Hypothesis}_2 \ldots)\]
  Meaning "For any substitution of logical variables in the Conclusion and Hypotheses, we may derive the conclusion if we can derive each of the hypotheses."

Example: Family Relations

• First, some facts with no hypotheses:
  \[
  \begin{align*}
  \text{(fact (parent george paul))} \\
  \text{(fact (parent martin george))} \\
  \text{(fact (parent martin martin_jr))} \\
  \text{(fact (parent martin donald))} \\
  \text{(fact (parent george ann))}
  \end{align*}
  \]
  \[
  \begin{align*}
  \text{(fact (ancestor ?X ?Y) (parent ?X ?Y))} \\
  \end{align*}
  \]
  - Intended meanings: May deduce that george is paul's parent, etc.
  - Now some general rules about relations:
    \[
    \begin{align*}
    \text{(fact (ancestor ?X ?Y) (parent ?X ?Y))} \\
    \end{align*}
    \]
  - Intended meanings:
    - For any values of ?X and ?Y, if we can deduce that ?X is ?Y's parent, then we may deduce that ?X is ?Y's ancestor.
    - For any values of ?X, ?Y, and ?Z, if we can deduce that ?X is ?Z's parent, and ?Z is ?Y's ancestor, then we may deduce that ?X is ?Z's ancestor.

Example, continued

\[
\begin{align*}
\text{(fact (parent george paul))} & \quad \text{(fact (parent martin george))} \\
\text{(fact (parent martin martin_jr))} & \quad \text{(fact (parent martin donald))} \\
\text{(fact (parent george ann))} \\
\end{align*}
\]

From these, we ought to be able to conclude that Martin is an ancestor of Ann, for example, or find out someone's ancestors or descendants.

Relations, Not Functions

• In this style of programming, we don't define functions, but rather relations.
  - Instead of saying \((\text{abs -3}) \Rightarrow 3\), we say \((\text{abs -3 3})\) (that is, "3 stands in the abs relation to -3.")
  - Instead of \((\text{add x y}) \Rightarrow z\), we say \((\text{add x y z})\).
  - This will allow us to run programs "both ways": from inputs to outputs, or from outputs to inputs.
Recap: A “Schemish” Prolog

- As a query, a Scheme expression, e.g., `(ordered (0 1 2))` represents a logical assertion.
  - Its top-level operator (e.g., `ordered`) names a predicate (true/false function).
  - Its operands are the data for this predicate: unlike Scheme programs, they don’t represent function calls—they are the literal data...
  - ...with the exception that logical variables, represented as symbols starting with ‘?’, stand for operands that may be replaced by other expressions.
- To define a predicate, we give rules for it:
  - `(fact CONCLUSION)` means that `CONCLUSION` is to be taken as true, for any replacement of its logical variables.
  - `(fact CONCLUSION HYPOTHESIS ...)` means that `CONCLUSION` is to be taken as true, assuming that the `HYPOTHESES` can all be shown to be true. Again, this is for all replacements of logical variables throughout the rule.

Operational and Declarative Meanings

- Thus,
  - means that for any replacement of ?P (e.g., 'brian') and ?F (e.g., 'potstickers') throughout the rule:
    - **Declarative Meaning**: If brian is hungry and has potstickers and likes potstickers, then brian will eat potstickers.
    - **Operational Meaning**: To show that brian will eat potstickers, show that brian is hungry, then that brian has potstickers, and then that brian likes potstickers.
- The **declarative meaning** allows us to look at our Scheme-Prolog program as a logical specification of a problem for which the system is to find a solution.
- The **operational meaning** allows us to look at our Scheme-Prolog specification as an executable program for searching for a solution.
- **Closed Universe Assumption**: We make only positive statements. The closest we come to saying that something is false is to say that we can’t prove it.

Demo

(Text of demos available in the files accompanying this lecture.)

How It’s Done (I): Unification

- In general, our system, given a target expression involving a predicate to prove, must find a fact that might assert that target, given a suitable replacement of logical variables.
- To do this, we try to pattern-match the conclusions of all our facts against the target expression.
- The pattern matching is called unification, [J. A. Robinson].
- For example, we say that `(likes brian potstickers)` unifies with the expression `(likes ?P ?F)`, if we substitute `brian` for `?P` and `potstickers` for `?F`.
- Might think of this substitution—called a unifier—as a Python dictionary mapping logical variables to expressions.
- Just for completeness, let’s take a look at one possible approach to creating these unifiers.
Unification (II)

- The substitution has to be uniform:
  - Can unify \( (\text{le } 0 \ 1) \) with \( (\text{le } ?X \ ?Y) \)
  - But cannot unify \( (\text{le } 0 \ 1) \) with \( (\text{le } ?X \ ?X) \)
- Everything is symmetric: if \( A \) unifies with \( B \), then \( B \) unifies with \( A \). Logical variables can appear in one or both.
- It is possible for logical variables to be unified with each other:
  Unify \( (\text{likes } ?P \ ?F) \) with \( (\text{likes } ?Q \ \text{potstickers}) \).
- We substitute \text{potstickers} for \( ?F \), and choose either to substitute \( ?Q \) for \( ?P \) or vice-versa.
- The result in either case means that any person likes potstickers.

Implementing Logical Variables and Substitutions

- A logical variable \( (?x) \) may be bound to any Scheme expression, including a logical variable.
- The set of all these bindings is called a unifier.
- Unifiers are like environments, but work a little differently.
- If \( ?x \) is bound to \( ?y \), then \( ?x \) is also bound to anything \( ?y \) is bound to.
- At that point, binding either \( ?x \) or \( ?y \) to something other than a logical variable binds both of them to that thing.
- Initially, every logical variable is bound to itself.

Implementing Logical Variables and Substitutions (II)

- Main operations on unifiers are \text{bind} and \text{binding}:

  ```python
class Unifier:
    def __init__(self, init={}):
      self.bindings = dict(init)  # Makes a copy

    def binding(self, expr):
      """Current binding of EXP. If EXP is not a logical variable, always returns EXP itself."""
      while expr in self.bindings:
        expr = self.bindings[expr]
      return expr

    def bind(self, var, value):
      assert is_logical_var(var)
      self.bindings[var] = value
```

- Can use ability to copy environments to \text{back out} of an attempted match.

Implementing Unification

A simple tree recursion with side-effects:

```python
def unify(E0, E1, unif):
    """Returns True iff E0 and E1 can be unified by an extension of UNIF. UNIF is modified to be this extension."""

def unify(E0, E1):
    E0 = unif.binding(E0); E1 = unif.binding(E1)
    if scheme_eqp(E0, E1): return True
    if is_logical_var(E0):
      unif.bind(E0, E1)  # E0 is always unbound here
      return True
    elif is_logical_var(E1):
      unif.bind(E1, E0)  # E1 is always unbound here
      return True
    elif scheme_atomp(E0) or scheme_atomp(E1): return False
    else:
      return unify(E0.first, E1.first) \ 
            and unify(E0.rest, E1.rest)
```

return unify(E0, E1)
Other Examples

- In the next lectures, we'll look at a couple of useful, but highly specialized declarative tools for programs that involve string manipulation.
- Regular expressions are patterns that describe strings. The system provides the means to match these patterns to strings.
- Backus-Naur Form (BNF) is another kind of pattern-matching facility that is more powerful than regular expressions. It is useful for describing things like programming languages in a human-readable format that also lends itself to actual language implementation.