Announcements
Efficient Sequence Processing
Sequence Operations
Sequence Operations

Map, filter, and reduce express sequence manipulation using compact expressions
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Example: Sum all primes in an interval from \( a \) (inclusive) to \( b \) (exclusive)
Sequence Operations

Map, filter, and reduce express sequence manipulation using compact expressions

Example: Sum all primes in an interval from a (inclusive) to b (exclusive)

```python
def sum_primes(a, b):
    total = 0
    x = a
    while x < b:
        if is_prime(x):
            total = total + x
        x = x + 1
    return total
```
Sequence Operations

Map, filter, and reduce express sequence manipulation using compact expressions

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Space: Constant
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Map, filter, and reduce express sequence manipulation using compact expressions

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```python
def sum_primes(a, b):
    return sum(filter(is_prime, range(a, b)))
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sum_primes(1, 6)
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Space: Constant

Also Constant
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def sum_primes(a, b):
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```

Demo

Space: \[\text{Constant}\]

Also Constant

(Demo)
Streams
Streams are Lazy Scheme Lists
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A stream is a list, but the rest of the list is computed only when needed:
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) \rightarrow 1
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) ⇒ 1

(cdr (cons 1 nil)) ⇒ ()
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A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) → 1

(cdr (cons 1 nil)) → ()

(cons 1 (cons 2 nil))
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\text{(car (cons 1 nil))} \rightarrow 1 \quad \text{(car (cons-stream 1 nil))} \rightarrow 1
\]

\[
\text{(cdr (cons 1 nil))} \rightarrow ()
\]

\[
\text{(cons 1 (cons 2 nil))}
\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) \rightarrow 1
(car (cons-stream 1 nil)) \rightarrow 1

(cdr (cons 1 nil)) \rightarrow ()
(cdr-stream (cons-stream 1 nil)) \rightarrow ()

(cons 1 (cons 2 nil))
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) → 1  (car (cons-stream 1 nil)) → 1
(cdr (cons 1 nil)) → ()  (cdr-stream (cons-stream 1 nil)) → ()
(cons 1 (cons 2 nil))  (cons-stream 1 (cons-stream 2 nil))
Streams are Lazy Scheme Lists

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(car (cons 1 nil)) -> 1
(car (cons-stream 1 nil)) -> 1
(cdr (cons 1 nil)) -> ()
(cdr-stream (cons-stream 1 nil)) -> ()
(cons 1 (cons 2 nil))
(cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:
Streams are Lazy Scheme Lists

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(cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:
(cons 1 (cons (/ 1 0) nil)) → ERROR
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil))  →  1
(cdr (cons 1 nil))  →  ()
(cons 1 (cons 2 nil))

(car (cons-stream 1 nil))  →  1
(cdr-stream (cons-stream 1 nil))  →  ()
(cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:

(cons 1 (cons (/ 1 0) nil))  →  ERROR
(cons-stream 1 (cons-stream (/ 1 0) nil))  →  (1 . #[promise (not forced)])
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
&\text{(car (cons 1 nil))} \rightarrow 1 \quad \text{(car (cons-stream 1 nil))} \rightarrow 1 \\
&\text{(cdr (cons 1 nil))} \rightarrow () \quad \text{(cdr-stream (cons-stream 1 nil))} \rightarrow () \\
&\text{(cons 1 (cons 2 nil))} \quad \text{(cons-stream 1 (cons-stream 2 nil))}
\end{align*}
\]

Errors only occur when expressions are evaluated:

\[
\begin{align*}
&\text{(cons 1 (cons (/ 1 0) nil))} \rightarrow \text{ERROR} \\
&\text{(cons-stream 1 (cons-stream (/ 1 0) nil))} \rightarrow (1 \ . \ #[\text{promise \ (not \ forced)]}) \\
&\text{(car (cons-stream 1 (cons-stream (/ 1 0) nil)))} \rightarrow 1
\end{align*}
\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 nil)) → 1  (car (cons-stream 1 nil)) → 1
(cdr (cons 1 nil)) → ()  (cdr-stream (cons-stream 1 nil)) → ()
(cons 1 (cons 2 nil)) (cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:

(cons 1 (cons (/ 1 0) nil)) → ERROR
(cons-stream 1 (cons-stream (/ 1 0) nil)) → (1 . #[promise (not forced)])
(car (cons-stream 1 (cons-stream (/ 1 0) nil))) → 1
(cdr-stream (cons-stream 1 (cons-stream (/ 1 0) nil))) → ERROR
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
\text{(car (cons 1 nil))} & \rightarrow 1 \\
\text{(cdr (cons 1 nil))} & \rightarrow () \\
\text{(cons 1 (cons 2 nil))} & \\
\end{align*}
\]

Errors only occur when expressions are evaluated:

\[
\begin{align*}
\text{(cons 1 (cons \((/ 1 0)\) nil))} & \rightarrow \text{ERROR} \\
\text{(cons-stream 1 (cons-stream \((/ 1 0)\) nil))} & \rightarrow (1 . \#[promise (not forced)]) \\
\text{(car (cons-stream 1 (cons-stream \((/ 1 0)\) nil)))} & \rightarrow 1 \\
\text{(cdr-stream (cons-stream 1 (cons-stream \((/ 1 0)\) nil)))} & \rightarrow \text{ERROR} \\
\text{(Demo)}
\end{align*}
\]
Stream Ranges are Implicit

A stream can give on-demand access to each element in order
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A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
      nil
      (cons-stream a (range-stream (+ a 1) b)))))
```
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

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```
scm> (car lots)
1
```
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```

```
scm> (car lots)
1
scm> (car (cdr-stream lots))
2
```
A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
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      (cons-stream a (range-stream (+ a 1) b))))

(define lots (range-stream 1 10000000000000000000))
```

```
scm> (car lots)
1
scm> (car (cdr-stream lots))
2
scm> (car (cdr-stream (cdr-stream lots)))
3
```
Infinite Streams
Integer Stream
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An integer stream is a stream of consecutive integers.
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The rest of the stream is not yet computed when the stream is created
Integer Stream

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The rest of the stream is not yet computed when the stream is created

\[
\text{(define (int-stream start)}
\text{(cons-stream start (int-stream (+ start 1))))}
\]
Integer Stream

An integer stream is a stream of consecutive integers

The rest of the stream is not yet computed when the stream is created

```
(define (int-stream start)
  (cons-stream start (int-stream (+ start 1))))
```
Stream Processing

(Demo)
Recursively Defined Streams
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The rest of a constant stream is the constant stream
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\((\text{define} \ \text{ones} \ (\text{cons-stream} \ 1 \ \text{ones}))\)
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))}
\]

\[
1 1 1 1 1 1 1 \ldots
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))} \quad 1\ 1\ 1\ 1\ 1\ 1\ \ldots
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
(\text{define \ ones \ (cons-stream \ 1 \ ones)})
\]

Combine two streams by separating each into car and cdr
Recursively Defined Streams

The rest of a constant stream is the constant stream

(define ones (cons-stream 1 ones))

1 1 1 1 1 1 ...

Combine two streams by separating each into car and cdr

(define (add-streams s t)
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))}
\]

Combine two streams by separating each into car and cdr

\[
\text{(define (add-streams s t)}
\]
\[
\text{(cons-stream (+ (car s) (car t))...)}
\]
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\text{(define (add-streams s t)}
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\text{\quad (cons-stream (+ (car s) (car t)))}
\]

\[
\text{\quad (add-streams (cdr-stream s) (cdr-stream t)))}
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Recursively Defined Streams

The rest of a constant stream is the constant stream

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\text{(define ones (cons-stream 1 ones))}
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Combine two streams by separating each into car and cdr

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\text{(define (add-streams s t)
  (cons-stream (+ (car s) (car t))
  (add-streams (cdr-stream s)
  (cdr-stream t))))}
\]

\[
\text{(define ints (cons-stream 1 (add-streams ones ints)))}
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

```
(define ones (cons-stream 1 ones))  1 1 1 1 1 1 ...
```

Combine two streams by separating each into car and cdr

```
(define (add-streams s t)
  (cons-stream (+ (car s) (car t))
               (add-streams (cdr-stream s)
                           (cdr-stream t))))
```

```
(define ints (cons-stream 1 (add-streams ones ints)))  1
```
Recursively Defined Streams

The rest of a constant stream is the constant stream

```
(define ones (cons-stream 1 ones))
```

Combine two streams by separating each into car and cdr

```
(define (add-streams s t)
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    (add-streams (cdr-stream s)
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Combine two streams by separating each into car and cdr

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\text{(define (add-streams s t)}
\text{ (cons-stream (+ (car s) (car t))}
\text{ (add-streams (cdr-stream s)}
\text{ (cdr-stream t)))))}
\]

\[
\text{(define ints (cons-stream 1 (add-streams ones ints))}
\text{ 1 1 1 1 1 1 ...}
\]

\[
\text{1 2 3 4 5 6 7 ...}
\]
Example: Repeats
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\[(\text{define } a (\text{cons-stream } 1 (\text{cons-stream } 2 (\text{cons-stream } 3 a))))\]
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))

What's (prefix a 8)? ( _ _ _ _ _ _ _ _)
Example: Repeats

\[
\begin{align*}
\text{Define } a & \, (\text{cons-stream} \ 1 \ (\text{cons-stream} \ 2 \ (\text{cons-stream} \ 3 \ a)))) \\
\text{Define } (f \ s) & \, (\text{cons-stream} \ (\text{car} \ s) \\
& \, \quad (\text{cons-stream} \ (\text{car} \ s) \\
& \, \quad \quad (f \ (\text{cdr-stream} \ s)))) \\
\text{What's } \text{prefix } a 8? & \, (\_ \ \_ \ \_ \ \_ \ \_ \ \_ \ \_ \ \_ \ \_ \ \_ \ \_)
\end{align*}
\]
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))

(define (f s) (cons-stream (car s)
                          (cons-stream (car s)
                                        (f (cdr-stream s)))))

What's (prefix a 8)? ( __ __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)? ( __ __ __ __ __ __ __ __ __ )
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
   (cons-stream (car s)
      (f (cdr-stream s))))))

(define (g s) (cons-stream (car s)
   (f (g (cdr-stream s))))))

What's (prefix a 8)?   ( __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)? ( __ __ __ __ __ __ __ __ )
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

What's (prefix a 8)?   ( __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)? ( __ __ __ __ __ __ __ __ )

What's (prefix (g a) 8)? ( __ __ __ __ __ __ __ __ )
Example: Repeats

\[
\text{(define } a \text{ (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))}
\]

\[
\text{(define } f \text{ s (cons-stream (car s)}
\quad (\text{cons-stream (car s)}
\quad (\text{f (cdr-stream s)})))\text{)}
\]

\[
\text{(define } g \text{ s (cons-stream (car s)}
\quad (\text{f (g (cdr-stream s)})))\text{)}
\]

What's (prefix a 8)? ( ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___)

What's (prefix (f a) 8)? ( ___ ___ ___ ___ ___ ___ ___ ___ ___ ___)

What's (prefix (g a) 8)? ( ___ ___ ___ ___ ___ ___ ___ ___ ___ ___)
Example: Repeats

\(\text{(define } a \text{ (cons-stream } 1 \text{ (cons-stream } 2 \text{ (cons-stream } 3 \text{ a)}))))\)

\(\text{(define } (f \text{ s}) \text{ (cons-stream (car } s)\text{ (cons-stream (car } s)\text{ (f (cdr-stream } s))))))\)

\(\text{(define } (g \text{ s}) \text{ (cons-stream (car } s)\text{ (f (g (cdr-stream } s))))))\)

What's \((\text{prefix } a \text{ 8})\)? \(\text{( _ _ _ _ _ _ _ _ _ _ _ _ _ _ )}\)

What's \((\text{prefix } (f \text{ a}) \text{ 8})\)? \(\text{( _ _ _ _ _ _ _ _ _ _ _ _ _ _ )}\)

What's \((\text{prefix } (g \text{ a}) \text{ 8})\)? \(\text{( _ _ _ _ _ _ _ _ _ _ _ _ _ _ )}\)
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))
```

```
(define (f s) (cons-stream (car s)
                         (cons-stream (car s)
                         (f (cdr-stream s)))))
```

```
(define (g s) (cons-stream (car s)
                         (f (g (cdr-stream s)))))
```

What's (prefix a 8)?

```
( 1 2 3 1 2 3 1 2 )
```

What's (prefix (f a) 8)?

```
( 1 1 1 1 1 1 1 1 )
```

What's (prefix (g a) 8)?

```
( _ _ _ _ _ _ _ _ )
```
Example: Repeats

\[(\text{define } a (\text{cons-stream } 1 (\text{cons-stream } 2 (\text{cons-stream } 3 a))))\]

\[(\text{define } (f \ s) (\text{cons-stream } (\text{car } s)\ (\text{cons-stream } (\text{car } s)\ (\text{f } (\text{cdr-stream } s))))))\]

\[(\text{define } (g \ s) (\text{cons-stream } (\text{car } s)\ (\text{f } (\text{g } (\text{cdr-stream } s))))))\]

What's (prefix a 8)?

\[(\ldots 1\ 2\ 3\ 1\ 2\ 3\ 1\ 2)\]

What's (prefix (f a) 8)?

\[(\ldots 1\ 1\ \ldots)\]

What's (prefix (g a) 8)?

\[(\ldots \ldots)\]
Example: Repeats

\[
\text{(define } a \ (\text{cons-stream } 1 \ (\text{cons-stream } 2 \ (\text{cons-stream } 3 \ a))))
\]

\[
\text{(define } f \ s \ (\text{cons-stream} \ (\text{car } s) \\
\quad \ (\text{cons-stream} \ (\text{car } s) \\
\quad \ (f \ (\text{cdr-stream } s)))))
\]

\[
\text{(define } g \ s \ (\text{cons-stream} \ (\text{car } s) \\
\quad \ (f \ (g \ (\text{cdr-stream } s)))))
\]

What's \(\text{(prefix } a \ 8)\)?
\[
\quad (1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2)
\]

What's \(\text{(prefix } (f \ a) \ 8)\)?
\[
\quad (1 \ 1 \ 2 \ _ \ _ \ _ \ _ \ _)
\]

What's \(\text{(prefix } (g \ a) \ 8)\)?
\[
\quad (_ \ _ \ _ \ _ \ _ \ _ \ _ \ _)
\]
Example: Repeats

\[
\text{(define } a \text{(cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))}
\]

\[
\text{(define } (f s) \text{(cons-stream (car s))}
\text{(cons-stream (car s))}
\text{(f (cdr-stream s)))))}
\]

\[
\text{(define } (g s) \text{(cons-stream (car s))}
\text{(f (g (cdr-stream s)))))}
\]

What's \((\text{prefix } a \text{ 8})\)?
\[
(1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2)
\]

What's \((\text{prefix } (f \ a) \text{ 8})\)?
\[
(1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2)
\]

What's \((\text{prefix } (g \ a) \text{ 8})\)?
\[
(1 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2)
\]
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))

(define (f s) (cons-stream (car s)
                             (cons-stream (car s)
                                          (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                             (f (g (cdr-stream s)))))

What's (prefix a 8)?  ( __  __  __  __  __  __  __  __  
                        1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)? ( __  __  __  __  __  __  __  __  
                          1 1 2 2 3 3 1 1 )

What's (prefix (g a) 8)? ( __  __  __  __  __  __  __  __  
                          1 2 2 3 3 1 1 )
Example: Repeats

```scheme
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s) (cons-stream (car s) (f (cdr-stream s))))))

(define (g s) (cons-stream (car s) (f (g (cdr-stream s))))))

What's (prefix a 8)? ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)? ( 1 1 2 2 3 3 1 1 )

What's (prefix (g a) 8)? ( 1 _ _ _ _ _ _ _ )
```
Example: Repeats

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What's (prefix a 8)?  ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)?  ( 1 1 2 2 3 3 1 1 )

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Example: Repeats

```scheme
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                            (f (g (cdr-stream s))))

What's (prefix a 8)?   ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)? ( 1 1 2 2 3 3 1 1 )

What's (prefix (g a) 8)? ( 1 2 2 3 3 1 1 )
```
Example: Repeats

\[
(\text{define } a \ (\text{cons-stream} \ 1 \ (\text{cons-stream} \ 2 \ (\text{cons-stream} \ 3 \ a))))
\]

\[
(\text{define } (f \ s) \ (\text{cons-stream} \ (\text{car} \ s) \\
\quad \ (\text{cons-stream} \ (\text{car} \ s) \\
\quad \quad \ (f \ (\text{cdr-stream} \ s)))))))
\]

\[
(\text{define } (g \ s) \ (\text{cons-stream} \ (\text{car} \ s) \\
\quad \ (f \ (g \ (\text{cdr-stream} \ s))))))
\]

What's (prefix a 8)? \[ (1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2) \]

What's (prefix (f a) 8)? \[ (1 \ 1 \ 2 \ 2 \ 3 \ 3 \ 1 \ 1) \]

What's (prefix (g a) 8)? \[ (1 \ 2 \ 2 \ 3 \ 3 \ 3 \ 3 \ 3) \]
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                          (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                          (f (g (cdr-stream s))))

What's (prefix a 8)?      ( __  __  __  __  __  __  __  __ )
What's (prefix (f a) 8)?  ( __  __  __  __  __  __  __  __ )
What's (prefix (g a) 8)?  ( __  __  __  __  __  __  __  __ )
```
Higher-Order Stream Functions
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream.
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream

(define (map f s)
  (if (null? s)
      nil
      (cons (f (car s))
            (map f
                 (cdr s))))))

(define (filter f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons (car s)
                (filter f (cdr s)))
          (filter f (cdr s))))))

(define (reduce f s start)
  (if (null? s)
      start
      (reduce f
              (cdr s)
              (f start (car s)))))
Higher-Order Functions on Streams

Implementations are identical, but change cons to \texttt{cons-stream} and change \texttt{cdr} to \texttt{cdr-stream}:

\begin{verbatim}
(define (map \textit{f} \textit{s})
  (if (null? \textit{s})
      nil
      (cons (f (car \textit{s}))
            (map \textit{f} (cdr \textit{s})))))

(define (filter \textit{f} \textit{s})
  (if (null? \textit{s})
      nil
      (if (f (car \textit{s}))
          (cons (car \textit{s})
                (filter \textit{f} (cdr \textit{s})))
          (filter \textit{f} (cdr \textit{s})))))

(define (reduce \textit{f} \textit{s} \textit{start})
  (if (null? \textit{s})
      \textit{start}
      (reduce \textit{f}
               \textit{f}
               (cdr \textit{s})
               (f \textit{start} (car \textit{s}))))
\end{verbatim}
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream.

```scheme
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
                  (map-stream f
                              (cdr-stream s))))
)

(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons-stream (car s)
                       (filter-stream f (cdr-stream s)))
          (filter-stream f (cdr-stream s))))
)

(define (reduce-stream f s start)
  (if (null? s)
      start
      (reduce-stream f
                     (cdr-stream s)
                     (f start (car s))))
)
```
A Stream of Primes
A Stream of Primes

For any prime $k$, any larger prime must not be divisible by $k$. 
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The stream of integers not divisible by any $k \leq n$ is:
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This recurrence is called the Sieve of Eratosthenes
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$2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$
A Stream of Primes

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\[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\]
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\[ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \]
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2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

(Demo)