Streams
Announcements
Efficient Sequence Processing
Sequence Operations
Sequence Operations

Map, filter, and reduce express sequence manipulation using compact expressions.
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Example: Sum all primes in an interval from \( a \) (inclusive) to \( b \) (exclusive)
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```python
def sum_primes(a, b):
    total = 0
    x = a
    while x < b:
        if is_prime(x):
            total = total + x
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Space: $\Theta(1)$
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def sum_primes(a, b):
    return sum(filter(is_prime, range(a, b)))

sum_primes(1, 6)
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    return sum(filter(is_prime, range(a, b)))
```

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(Demo)
Streams
Streams are Lazy Scheme Lists
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[(\text{car (cons 1 2)}) \rightarrow 1\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) \rightarrow 1

(cdr (cons 1 2)) \rightarrow 2
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) \rightarrow 1
(cdr (cons 1 2)) \rightarrow 2
(cons 1 (cons 2 nil))
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) → 1        (car (cons-stream 1 2)) → 1
(cdr (cons 1 2)) → 2
(cons 1 (cons 2 nil))
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
(\text{car } (\text{cons } 1 2)) & \rightarrow 1 & (\text{car } (\text{cons-stream } 1 2)) & \rightarrow 1 \\
(\text{cdr } (\text{cons } 1 2)) & \rightarrow 2 & (\text{cdr-stream } (\text{cons-stream } 1 2)) & \rightarrow 2 \\
(\text{cons } 1 (\text{cons } 2 \text{ nil})) & & & 
\end{align*}
\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) → 1          (car (cons-stream 1 2)) → 1
(cdr (cons 1 2)) → 2          (cdr-stream (cons-stream 1 2)) → 2
(cons 1 (cons 2 nil))         (cons-stream 1 (cons-stream 2 nil))
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
\text{(car (cons 1 2))} & \rightarrow 1 & \text{(car (cons-stream 1 2))} & \rightarrow 1 \\
\text{(cdr (cons 1 2))} & \rightarrow 2 & \text{(cdr-stream (cons-stream 1 2))} & \rightarrow 2 \\
\text{(cons 1 (cons 2 nil))} & \rightarrow \text{Error} & \text{(cons-stream 1 (cons-stream 2 nil))} & \rightarrow \text{Error}
\end{align*}
\]

Errors only occur when expressions are evaluated:
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[(\text{car } (\text{cons } 1 \ 2)) \rightarrow 1\]  \[(\text{car } (\text{cons}-\text{stream } 1 \ 2)) \rightarrow 1\]
\[(\text{cdr } (\text{cons } 1 \ 2)) \rightarrow 2\]  \[(\text{cdr}-\text{stream } (\text{cons}-\text{stream } 1 \ 2)) \rightarrow 2\]
\[(\text{cons } 1 \ (\text{cons } 2 \ \text{nil}))\]  \[(\text{cons}-\text{stream } 1 \ (\text{cons}-\text{stream } 2 \ \text{nil}))\]

Errors only occur when expressions are evaluated:

\[(\text{cons } 1 \ (/ \ 1 \ 0)) \rightarrow \text{ERROR}\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
\text{(car (cons 1 2))} & \rightarrow 1 & \text{(car (cons-stream 1 2))} & \rightarrow 1 \\
\text{(cdr (cons 1 2))} & \rightarrow 2 & \text{(cdr-stream (cons-stream 1 2))} & \rightarrow 2 \\
\text{(cons 1 (cons 2 nil))} & \rightarrow \text{ERROR} & \text{(cons-stream 1 (cons-stream 2 nil))} & \rightarrow \text{ERROR}
\end{align*}
\]

Errors only occur when expressions are evaluated:

\[
\begin{align*}
\text{(cons 1 (/ 1 0))} & \rightarrow \text{ERROR} \\
\text{(car (cons 1 (/ 1 0)))} & \rightarrow \text{ERROR}
\end{align*}
\]
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A stream is a list, but the rest of the list is computed only when needed:

\[
\begin{align*}
\text{car (cons 1 2)} & \rightarrow 1 & \text{car (cons-stream 1 2)} & \rightarrow 1 \\
\text{cdr (cons 1 2)} & \rightarrow 2 & \text{cdr-stream (cons-stream 1 2)} & \rightarrow 2 \\
(\text{cons 1 (cons 2 nil)}) & & \text{cons-stream 1 (cons-stream 2 nil)}
\end{align*}
\]

Errors only occur when expressions are evaluated:

\[
\begin{align*}
\text{cons 1 (/ 1 0)} & \rightarrow \text{ERROR} \\
\text{car (cons 1 (/ 1 0))} & \rightarrow \text{ERROR} \\
\text{cdr (cons 1 (/ 1 0))} & \rightarrow \text{ERROR}
\end{align*}
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Streams are Lazy Scheme Lists

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\text{(car (cons 1 2))} & \rightarrow 1 \\
\text{(cdr (cons 1 2))} & \rightarrow 2 \\
\text{(cons 1 (cons 2 nil))} & \\
\end{align*}
\]

\[
\begin{align*}
\text{(car (cons-stream 1 2))} & \rightarrow 1 \\
\text{(cdr-stream (cons-stream 1 2))} & \rightarrow 2 \\
\text{(cons-stream 1 (cons-stream 2 nil))} & \\
\end{align*}
\]

Errors only occur when expressions are evaluated:

\[
\begin{align*}
\text{(cons 1 (/ 1 0))} & \rightarrow \text{ERROR} \\
\text{(car (cons 1 (/ 1 0)))} & \rightarrow \text{ERROR} \\
\text{(cdr (cons 1 (/ 1 0)))} & \rightarrow \text{ERROR} \\
\end{align*}
\]
Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) $\rightarrow$ 1  \hspace{1cm} (car (cons-stream 1 2)) $\rightarrow$ 1

(cdr (cons 1 2)) $\rightarrow$ 2  \hspace{1cm} (cdr-stream (cons-stream 1 2)) $\rightarrow$ 2

(cons 1 (cons 2 nil)) \hspace{1cm} (cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:

(cons 1 (/ 1 0)) $\rightarrow$ ERROR  \hspace{1cm} (cons-stream 1 (/ 1 0)) $\rightarrow$ (1 . #[promise (not forced)])

(car (cons 1 (/ 1 0))) $\rightarrow$ ERROR  \hspace{1cm} (car (cons-stream 1 (/ 1 0))) $\rightarrow$ 1

(cdr (cons 1 (/ 1 0))) $\rightarrow$ ERROR
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A stream is a list, but the rest of the list is computed only when needed:

(car (cons 1 2)) → 1  (car (cons-stream 1 2)) → 1
(cdr (cons 1 2)) → 2  (cdr-stream (cons-stream 1 2)) → 2
(cons 1 (cons 2 nil)) (cons-stream 1 (cons-stream 2 nil))

Errors only occur when expressions are evaluated:

(cons 1 (/ 1 0)) → ERROR  (cons-stream 1 (/ 1 0)) → (1 . #[promise (not forced)])
(car (cons 1 (/ 1 0))) → ERROR  (car (cons-stream 1 (/ 1 0))) → 1
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Streams are Lazy Scheme Lists

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```
(car (cons 1 2)) -> 1  (car (cons-stream 1 2)) -> 1
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(cons 1 (cons 2 nil))  (cons-stream 1 (cons-stream 2 nil))
```

Errors only occur when expressions are evaluated:

```
(cons 1 (/ 1 0))   -> ERROR  (cons-stream 1 (/ 1 0))   -> (1 . #[promise (not forced)])
(car (cons 1 (/ 1 0))) -> ERROR  (car (cons-stream 1 (/ 1 0))) -> 1
(cdr (cons 1 (/ 1 0))) -> ERROR  (cdr-stream (cons-stream 1 (/ 1 0))) -> ERROR
```

(Demo)
Stream Ranges are Implicit

A stream can give on-demand access to each element in order
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
      nil
      (cons-stream a (range-stream (+ a 1) b)))))
```
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

\[
(\text{define} \ (\text{range-stream} \ a \ b) \\
(\text{if} \ (\geq a \ b) \\
 \quad \text{nil} \\
 \quad (\text{cons-stream} \ a \ (\text{range-stream} \ (+ a 1) \ b))) \\
(\text{define} \ \text{lots} \ (\text{range-stream} \ 1 \ 10000000000000000000))
\]
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
      nil
      (cons-stream a (range-stream (+ a 1) b)))))

(define lots (range-stream 1 10000000000000000000))

(scm> (car lots)
1
```
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

\[
\begin{align*}
\text{(define} & \text{(range-stream } a \text{ } b) \\
& \text{(if} \ (\geq \ a \ b) \\
& \quad \text{nil} \\
& \quad \text{(cons-stream} \ a \ (\text{range-stream} \ (+ \ a \ 1) \ b))))
\end{align*}
\]

\[
(\text{define} \ \text{lots} \ (\text{range-stream} \ 1 \ 10000000000000000000))
\]

```
scm> (car lots)
1
scm> (car (cdr-stream lots))
2
```
Stream Ranges are Implicit

A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
      nil
      (cons-stream a (range-stream (+ a 1) b))))

(define lots (range-stream 1 10000000000000000000))
```

```scm
scm> (car lots)
1
scm> (car (cdr-stream lots))
2
scm> (car (cdr-stream (cdr-stream lots)))
3
```
Infinite Streams
Integer Stream
An integer stream is a stream of consecutive integers.
**Integer Stream**

An integer stream is a stream of consecutive integers.

The rest of the stream is not yet computed when the stream is created.
Integer Stream

An integer stream is a stream of consecutive integers

The rest of the stream is not yet computed when the stream is created

```
(define (int-stream start)
  (cons-stream start (int-stream (+ start 1))))
```
Integer Stream

An integer stream is a stream of consecutive integers
The rest of the stream is not yet computed when the stream is created

*(define (int-stream start)*
  *(cons-stream start (int-stream (+ start 1)))*)

(Demo)
Stream Processing

(Demo)
Recursively Defined Streams
Recursively Defined Streams

The rest of a constant stream is the constant stream
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[(\text{define } \text{ones } (\text{cons-stream } 1 \text{ ones}))\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))}
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

```
(define ones (cons-stream 1 ones))  1 1 1 1 1 1 ...
```
Recursively Defined Streams

The rest of a constant stream is the constant stream

(define ones (cons-stream 1 ones))

Combine two streams by separating each into car and cdr
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[ \text{(define ones (cons-stream 1 ones))} \]

Combine two streams by separating each into car and cdr

\[ \text{(define (add-streams s t)} \]

1 1 1 1 1 1 1 ...
Recursively Defined Streams

The rest of a constant stream is the constant stream

```
(define ones (cons-stream 1 ones))
```

1 1 1 1 1 1 ...

Combine two streams by separating each into car and cdr

```
(define (add-streams s t)
  (cons-stream (+ (car s) (car t)))
```

Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))}
\]

Combine two streams by separating each into car and cdr

\[
\text{(define (add-streams s t)}
\text{ (cons-stream (+ (car s) (car t))}
\text{ (add-streams (cdr-stream s) (cdr-stream t))))}
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[(\text{define} \ \text{ones} \ (\text{cons-stream} \ 1 \ \text{ones}))\]

Combine two streams by separating each into car and cdr

\[(\text{define} \ (\text{add-streams} \ s \ t) \ (\text{cons-stream} \ (+ \ (\text{car} \ s) \ (\text{car} \ t)) \ (\text{add-streams} \ (\text{cdr-stream} \ s) \ (\text{cdr-stream} \ t))))\]

\[(\text{define} \ \text{ints} \ (\text{cons-stream} \ 1 \ (\text{add-streams} \ \text{ones} \ \text{ints})))\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (\text{cons-stream} 1 \text{ ones}))}
\]

Combine two streams by separating each into \text{car} and \text{cdr}

\[
\text{(define (add-streams s t)}
\text{\text{\text{\text{(cons-stream (}}\text{+ (}}\text{\text{car s) (}}\text{\text{car t))}}
\text{\text{\text{\text{(add-streams (}}\text{cdr-stream s) (}}\text{\text{cdr-stream t)})))}}
\]

\[
\text{(define \text{ints (cons-stream 1 (add-streams ones ints))}}
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[
\text{(define ones (cons-stream 1 ones))}
\]

Combine two streams by separating each into \text{car} and \text{cdr}

\[
\text{(define (add-streams s t)}
\text{ (cons-stream (+ (car s) (car t))}
\text{ (add-streams (cdr-stream s) (cdr-stream t))})
\]

\[
\text{(define ints (cons-stream 1 (add-streams ones ints))})
\]
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\text{\quad (add-streams (cdr-stream s) (cdr-stream t))})
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\]

\[
\text{(define ints (cons-stream 1 (add-streams ones ints)))}
\]
Recursively Defined Streams

The rest of a constant stream is the constant stream

\[(\text{define} \ \text{ones} \ (\text{cons-stream} \ 1 \ \text{ones}))\]

Combine two streams by separating each into car and cdr

\[(\text{define} \ (\text{add-streams} \ s \ t) \n \quad (\text{cons-stream} \ (+ \ (\text{car} \ s) \ (\text{car} \ t)) \n \quad \ (\text{add-streams} \ (\text{cdr-stream} \ s) \n \quad \ (\text{add-streams} \ (\text{cdr-stream} \ t))))\n\]

\[(\text{define} \ \text{ints} \ (\text{cons-stream} \ 1 \ (\text{add-streams} \ \text{ones} \ \text{ints})))\]
Example: Repeats
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

What's (prefix a 8)?

( __ __ __ __ __ __ __ )
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                            (cons-stream (car s)
                                          (f (cdr-stream s)))))

What's (prefix a 8)?      ( __ __ __ __ __ __ __ __ )
Example: Repeats

\[
\begin{align*}
&\text{(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))} \\
&\text{(define (f s) (cons-stream (car s) (cons-stream (car s) (f (cdr-stream s)))))}
\end{align*}
\]

What's (prefix a 8)? \hfill ( __ __ __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)? \hfill ( __ __ __ __ __ __ __ __ __ __ )
Example: Repeats

\[(\text{define } a (\text{cons-stream} \ 1 \ (\text{cons-stream} \ 2 \ (\text{cons-stream} \ 3 \ a))))\]

\[(\text{define } (f \ s) (\text{cons-stream} \ (\text{car} \ s)) \ (\text{cons-stream} \ (\text{car} \ s) \ (f \ (\text{cdr-stream} \ s))))\]

\[(\text{define } (g \ s) (\text{cons-stream} \ (\text{car} \ s) \ (f \ (g \ (\text{cdr-stream} \ s))))\)]

What's \((\text{prefix } a \ 8)\)?  ( __ __ __ __ __ __ __ __ )

What's \((\text{prefix } (f \ a) \ 8)\)?  ( __ __ __ __ __ __ __ __ )
Example: Repeats

\[
\text{(define } a \, (\text{cons-stream } 1 \, (\text{cons-stream } 2 \, (\text{cons-stream } 3 \, a))))
\]

\[
\text{(define } f\, s) \, (\text{cons-stream} \, \text{(car } s) \\
\text{ (cons-stream} \, \text{(car } s) \\
\text{ (f} \, \text{(cdr-stream } s)))
\]

\[
\text{(define } g\, s) \, (\text{cons-stream} \, \text{(car } s) \\
\text{ (f} \, \text{(g} \, \text{(cdr-stream } s))))
\]

What's (prefix a 8)?  ( __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)?  ( __ __ __ __ __ __ __ __ )

What's (prefix (g a) 8)?  ( __ __ __ __ __ __ __ __ )
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))
```

```
(define (f s) (cons-stream (car s)
                       (cons-stream (car s)
                         (f (cdr-stream s))))))
```

```
(define (g s) (cons-stream (car s)
                       (f (g (cdr-stream s)))))
```

What's (prefix a 8)?    ( __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)? ( __ __ __ __ __ __ __ __ )

What's (prefix (g a) 8)? ( __ __ __ __ __ __ __ __ )
Example: Repeats

\[
(\text{define } a (\text{cons-stream } 1 (\text{cons-stream } 2 (\text{cons-stream } 3 a))))
\]

\[
(\text{define } (f s) (\text{cons-stream } (\text{car } s) \\
(\text{cons-stream } (\text{car } s) \\
(\text{f } (\text{cdr-stream } s))))))
\]

\[
(\text{define } (g s) (\text{cons-stream } (\text{car } s) \\
(f (g (\text{cdr-stream } s))))))
\]

What's (prefix a 8)?  ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)?  ( _ _ _ _ _ _ _ _ )

What's (prefix (g a) 8)?  ( _ _ _ _ _ _ _ _ )
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))

(define (f s) (cons-stream (car s)
  (cons-stream (car s)
    (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
  (f (g (cdr-stream s)))))
```

What's (prefix a 8)? ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)? ( 1 _ _ _ _ _ _ _ )

What's (prefix (g a) 8)? ( _ _ _ _ _ _ _ _ _ )
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a)))))

(define (f s) (cons-stream (car s) (cons-stream (car s) (f (cdr-stream s)))))

(define (g s) (cons-stream (car s) (f (g (cdr-stream s)))))

What's (prefix a 8)?  ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?  ( 1 1 __ __ __ __ __ __ )

What's (prefix (g a) 8)?  ( __ __ __ __ __ __ __ __ )
Example: Repeats

\[
\begin{align*}
&\text{(define } a \text{ (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))} \\
&(\text{define } (f \ s) \text{ (cons-stream (car } s\text{) (cons-stream (car } s\text{) (f (cdr-stream } s\text{))))}) \\
&(\text{define } (g \ s) \text{ (cons-stream (car } s\text{) (f (g (cdr-stream } s\text{))))})
\end{align*}
\]

What's \( \text{(prefix } a \text{ 8)?} \) \( \text{(1 2 3 1 2 3 1 2)} \)

What's \( \text{(prefix } (f \ a) \text{ 8)?} \) \( \text{(1 1 2)} \)

What's \( \text{(prefix } (g \ a) \text{ 8)?} \) \( \text{(1 2)} \)

What's \( \text{(prefix } (g \ a) \text{ 8)?} \) \( \text{(1 2)} \)
Example: Repeats

\[
(\text{define } a (\text{cons-stream } 1 (\text{cons-stream } 2 (\text{cons-stream } 3 a))))
\]

\[
(\text{define } (f \ s) (\text{cons-stream } (\text{car } s) \\
(\text{cons-stream } (\text{car } s) \\
(f (\text{cdr-stream } s))))))
\]

\[
(\text{define } (g \ s) (\text{cons-stream } (\text{car } s) \\
(f (g (\text{cdr-stream } s))))))
\]

What's (prefix a 8)?  \( (\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ ) \)

What's (prefix (f a) 8)?  \( (1 \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ ) \)

What's (prefix (g a) 8)?  \( (\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ ) \)

What's (prefix (g a) 8)?  \( (\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ ) \)
Example: Repeats

\[(define \ a \ (cons-stream \ 1 \ (cons-stream \ 2 \ (cons-stream \ 3 \ a))))\]

\[(define \ (f \ s) \ (cons-stream \ (car \ s) \ (cons-stream \ (car \ s) \ (f \ (cdr-stream \ s)))))\]

\[(define \ (g \ s) \ (cons-stream \ (car \ s) \ (f \ (g \ (cdr-stream \ s))))))\]

What's (prefix a 8)? \( (\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ )\)

What's (prefix (f a) 8)? \( (1 \ 1 \ 2 \ 2 \ 3 \ 3 \ 1 \ 1 \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ )\)

What's (prefix (g a) 8)? \( (\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ )\)
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                            (cons-stream (car s)
                             (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                            (f (g (cdr-stream s)))))

What's (prefix a 8)? ( _ _ _ _ _ _ _ _ _ _)

What's (prefix (f a) 8)? ( _ _ _ 1 2 _ _ _ _ _ _ _ _ _ _ _ _)

What's (prefix (g a) 8)? ( _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                            (cons-stream (car s)
                                         (f (cdr-stream s))))))

(define (g s) (cons-stream (car s)
                            (f (g (cdr-stream s))))))

What's (prefix a 8)?  ( 1 2 3 1 2 3 1 2 )

What's (prefix (f a) 8)? ( 1 1 2 2 3 3 1 1 )

What's (prefix (g a) 8)? ( 1 2 2 1 1 )
```
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                            (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))

What's (prefix a 8)?  ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?  ( __  __  __  __  __  __  __  __ )

What's (prefix (g a) 8)?  ( __  __  __  __  __  __  __  __ )
Example: Repeats

(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

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  (cons-stream (car s)
    (f (cdr-stream s))))))

(define (g s) (cons-stream (car s)
  (f (g (cdr-stream s))))))

What's (prefix a 8)?
  ( __ __ __ __ __ __ __ __
    1 2 3 1 2 3 1 2)

What's (prefix (f a) 8)?
  ( __ __ __ __ __ __ __ __
    1 1 2 2 3 3 1 1)

What's (prefix (g a) 8)?
  ( __ __ __ __ __ __ __ __
    1 2 2 3 3 3 3 __)
Example: Repeats

```scheme
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                             (cons-stream (car s)
                                          (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                            (f (g (cdr-stream s)))))
```

What's (prefix a 8)?  
( __ __ __ __ __ __ __ __ )

What's (prefix (f a) 8)?  
( __ __ __ __ __ __ __ __ )

What's (prefix (g a) 8)?  
( __ __ __ __ __ __ __ __ )
Higher-Order Stream Functions
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream.
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream
and change cdr to cdr-stream

(define (map f s)
  (if (null? s)
      nil
      (cons (f (car s))
            (map f
                 (cdr s)))))

(define (filter f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons (car s)
                (filter f (cdr s)))
          (filter f (cdr s)))))

(define (reduce f s start)
  (if (null? s)
      start
      (reduce f
              (cdr s)
              (f start (car s)))))

:%s/\s(map|filter|reduce|cdr|cons)/\1-stream/g
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream

\[
\begin{align*}
\text{(define (map f s)} &= \text{if (null? s)} \\
&\quad \text{nil} \\
&\quad \text{(cons (f (car s)} \\
&\quad \text{(map f (cdr s))}) \\
\text{(define (filter f s)} &= \text{if (null? s)} \\
&\quad \text{nil} \\
&\quad \text{(if (f (car s)} \\
&\quad \text{(cons (car s)} \\
&\quad \text{(filter f (cdr s))}) \\
&\quad \text{(filter f (cdr s))}) \\
\text{(define (reduce f s start)} &= \text{if (null? s)} \\
&\quad \text{start} \\
&\quad \text{(reduce f (cdr s)} \\
&\quad \text{(f start (car s))})
\end{align*}
\]
Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream.

```scheme
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
                   (map-stream f
                                (cdr-stream s))))
)

(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
      (cons-stream (car s)
                   (filter-stream f (cdr-stream s))
                   (filter-stream f (cdr-stream s))))
)

(define (reduce-stream f s start)
  (if (null? s)
      start
      (reduce-stream f
                     (cdr-stream s)
                     (f start (car s))))
)
```
A Stream of Primes
A Stream of Primes

For any prime $k$, any larger prime must not be divisible by $k$. 
A Stream of Primes

For any prime $k$, any larger prime must not be divisible by $k$.

The stream of integers not divisible by any $k \leq n$ is:
A Stream of Primes

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The stream of integers not divisible by any $k \leq n$ is:
• The stream of integers not divisible by any $k < n$
A Stream of Primes

For any prime $k$, any larger prime must not be divisible by $k$.

The stream of integers not divisible by any $k \leq n$ is:
• The stream of integers not divisible by any $k < n$
• Filtered to remove any element divisible by $n$
A Stream of Primes

For any prime \( k \), any larger prime must not be divisible by \( k \).

The stream of integers not divisible by any \( k \leq n \) is:
• The stream of integers not divisible by any \( k < n \)
• Filtered to remove any element divisible by \( n \)
This recurrence is called the Sieve of Eratosthenes
A Stream of Primes

For any prime k, any larger prime must not be divisible by k.

The stream of integers not divisible by any k ≤ n is:
• The stream of integers not divisible by any k < n
• Filtered to remove any element divisible by n
This recurrence is called the Sieve of Eratosthenes

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
A Stream of Primes

For any prime \( k \), any larger prime must not be divisible by \( k \).

The stream of integers not divisible by any \( k \leq n \) is:
• The stream of integers not divisible by any \( k < n \)
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A Stream of Primes

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\[ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \]
A Stream of Primes

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\[ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \]
A Stream of Primes

For any prime \( k \), any larger prime must not be divisible by \( k \).

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\[ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \]
A Stream of Primes

For any prime $k$, any larger prime must not be divisible by $k$.

The stream of integers not divisible by any $k \leq n$ is:
- The stream of integers not divisible by any $k < n$
- Filtered to remove any element divisible by $n$

This recurrence is called the Sieve of Eratosthenes

\[
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
\]

(Demo)
Promises
Implementing Streams with Delay and Force
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it. Delaying an expression creates a promise to evaluate it later in the current environment. Forcing a promise returns its value in the environment in which it was defined.
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```scm
(define promise (let ((x 2)) (delay (+ x 1)) ))
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1))))
```

```
scm> (force promise)
3
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it. Delaying an expression creates a promise to evaluate it later in the current environment. Forcing a promise returns its value in the environment in which it was defined.

```sml
(define promise (let ((x 2)) (delay (+ x 1))))

(define x 5)
(= (force promise) 3)
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1))))

scm> (define x 5)

scm> (force promise)
3
```

```
(define-macro (delay expr) `(lambda () ,expr))
(define (force promise) (promise))
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1))))
(define promise (let ((x 2)) (lambda () (+ x 1))))

scm> (define x 5)
```

```
scm> (force promise)
```

```
3
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it. Delaying an expression creates a promise to evaluate it later in the current environment. Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1))))
(define promise (let ((x 2)) (lambda () (+ x 1))))

scm> (define x 5)

scm> (force promise)
3
```

A stream is a list, but the rest of the list is computed only when forced.
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it. Delaying an expression creates a promise to evaluate it later in the current environment. Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1))))
(define promise (let ((x 2)) (lambda () (+ x 1))))
scm> (define x 5)
scm> (force promise)
3
```

A stream is a list, but the rest of the list is computed only when **forced**:

```
scm> (define ones (cons-stream 1 ones))
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```
scm> (define promise (let ((x 2)) (delay (+ x 1)))
   (define promise (let ((x 2)) (lambda () (+ x 1))))
scm> (define x 5)
scm> (force promise)
3
```

A stream is a list, but the rest of the list is computed only when forced:

```
scm> (define ones (cons-stream 1 ones))
(1 . #[promise (not forced)])
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```scheme
(define promise (let ((x 2)) (delay (+ x 1))))
(define promise (let ((x 2)) (lambda () (+ x 1))))
```

```scheme
(define x 5)
```

```scheme
(force promise)
```

A stream is a list, but the rest of the list is computed only when forced:

```scheme
(define ones (cons-stream 1 ones))
```

```scheme
(define ones (cons-stream 1 ones))
```

A stream is a list, but the rest of the list is computed only when forced:

```
(define-macro (cons-stream a b) `(cons ,a (delay ,b)))
(define (cdr-stream s) (force (cdr s)))
```
Implementing Streams with Delay and Force

A promise is an expression, along with an environment in which to evaluate it.

Delaying an expression creates a promise to evaluate it later in the current environment.

Forcing a promise returns its value in the environment in which it was defined.

```
(define promise (let ((x 2)) (delay (+ x 1))))
(define promise (let ((x 2)) (lambda () (+ x 1))))
```

```
(define x 5)
```

```
(force promise)
```

A stream is a list, but the rest of the list is computed only when forced:

```
(define ones (cons-stream 1 ones))
```

```
(define-macro (cons-stream a b) `(cons ,a (delay ,b)))
```

```
(define (cdr-stream s) (force (cdr s)))
```