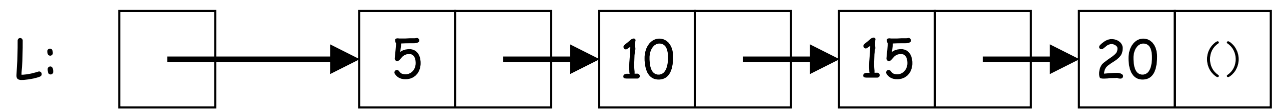
Lecture #36: Review Linked Lists and Trees

Linked-List Characteristics



- Recursive structure: a linked list (L) is a sequence of nodes that contain a data item (L.first) + reference to a linked list (L.rest). A special value (Link.empty) represents an empty list.
- Suggests that operations on it can be formulated as a tail recursion, or, equivalently, as an iteration.
- Complexity:
  - Fetch item or node #k for constant k: $\Theta(1)$.
  - Add node to the front: $\Theta(1)$.
  - Add a node after a node, M, in the middle of a list, assuming we have already found M: $\Theta(1)$.
  - Fetch item or node #k for arbitrary k: $\Theta(k)$ or (since $k < N$ for N the length of the list) $\Theta(N)$ in the worst case.
  - Find length of list or find a data item in the list: $\Theta(N)$ worst case.

Linked-List Class

```
class Link:
    """A linked list node."""
    empty = ()
    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest
    def __repr__(self):
        """Return string denotation of SELF as Link(first, rest)."""
    def __str__(self):
        """Return string denotation of SELF as <item item ...>."""

def toLinked(L):
    """Returns a linked-list representation of the Python iterable L."""
    if len(L) == 0:
        return Link.empty
    result = last = Link(L[0], Link.empty)
    for item in L[1:]:
        last.rest = Link(item)
        last = last.rest
    return result
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    return result
#### Can you make this work for nested Python lists? ####
```

Exercise: Find Node k in List

```
def split(L):
    """Returns (Mid, Last, Length), where Last is the last node in
    linked list L, Mid is the node at or (for even length) just before
    the middle, and Length is the length."""
```

- Do this with *one pass* through L, with constant extra space (i.e., iteratively with no auxiliary lists or other containers).

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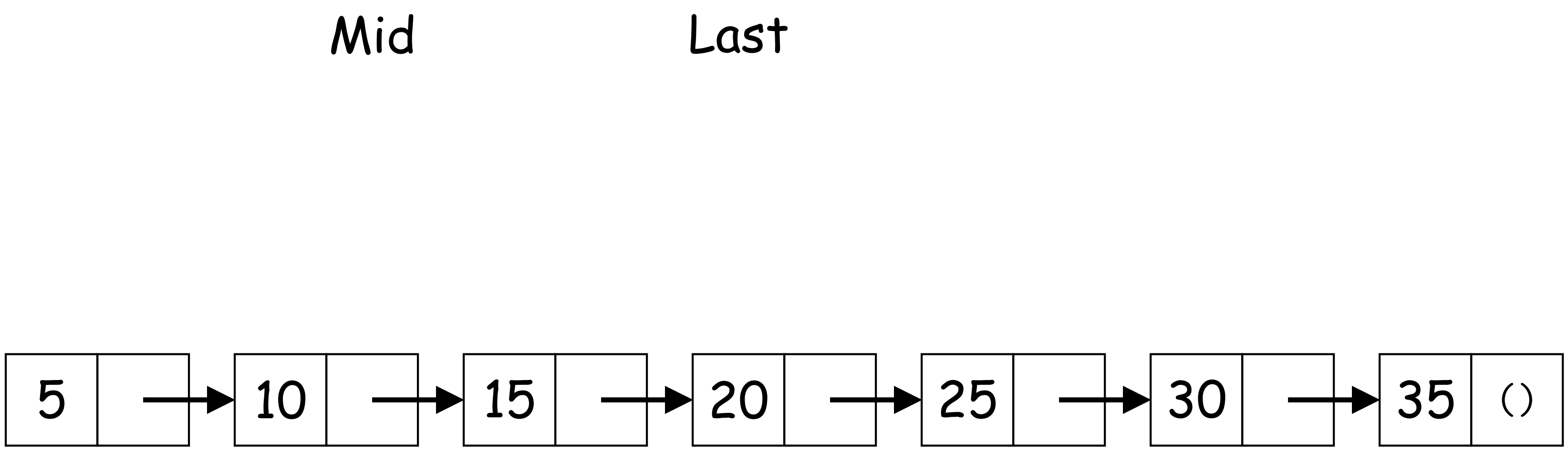

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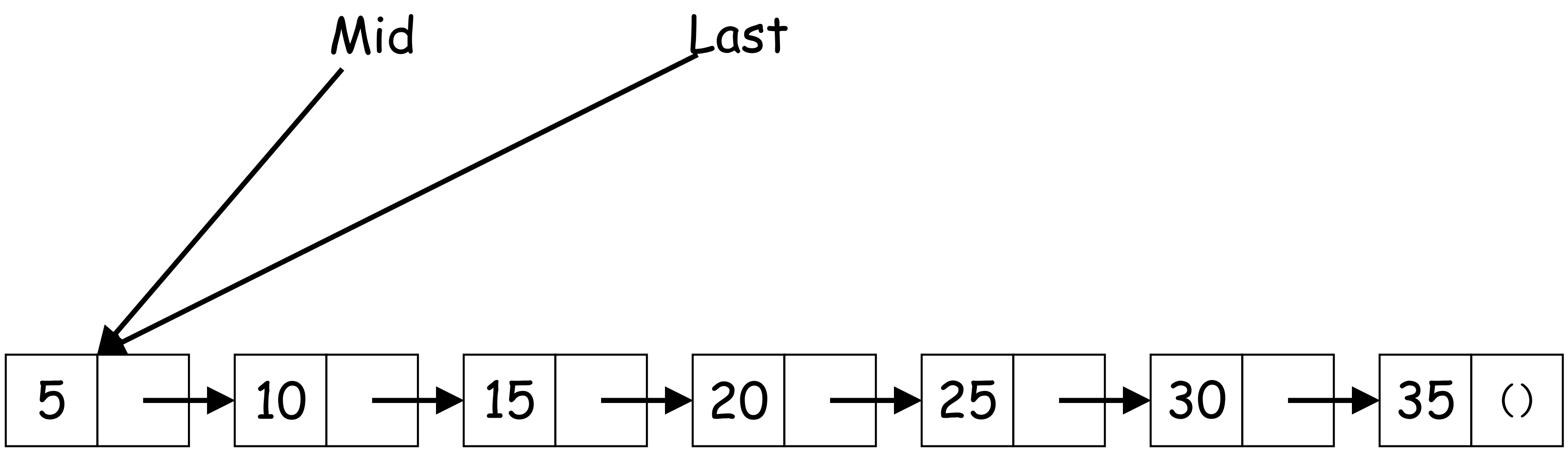

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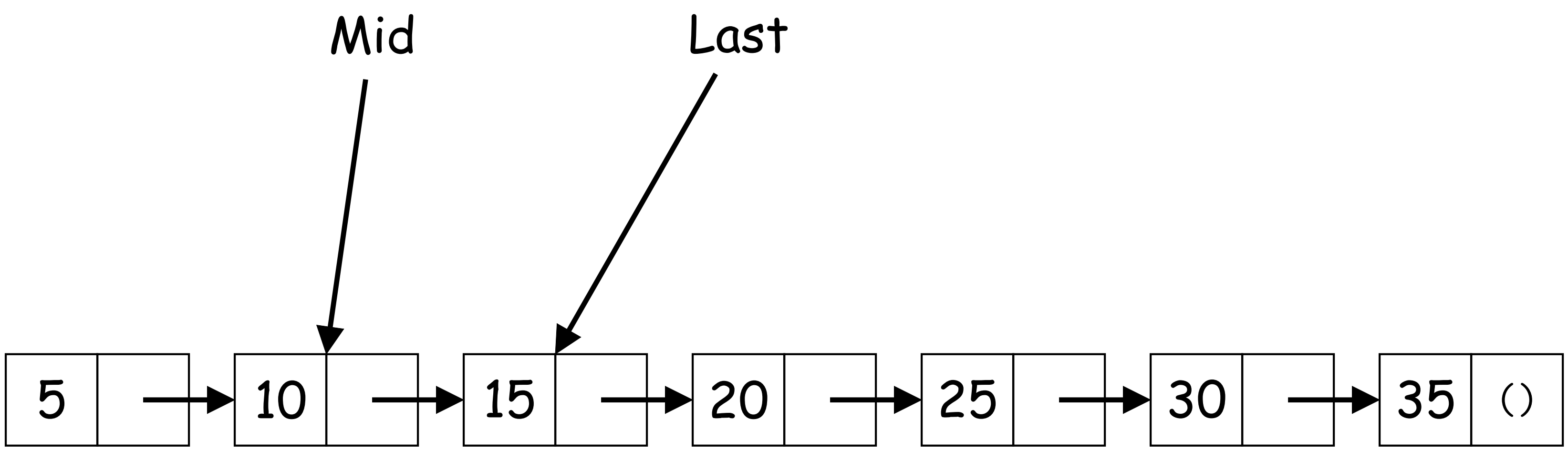

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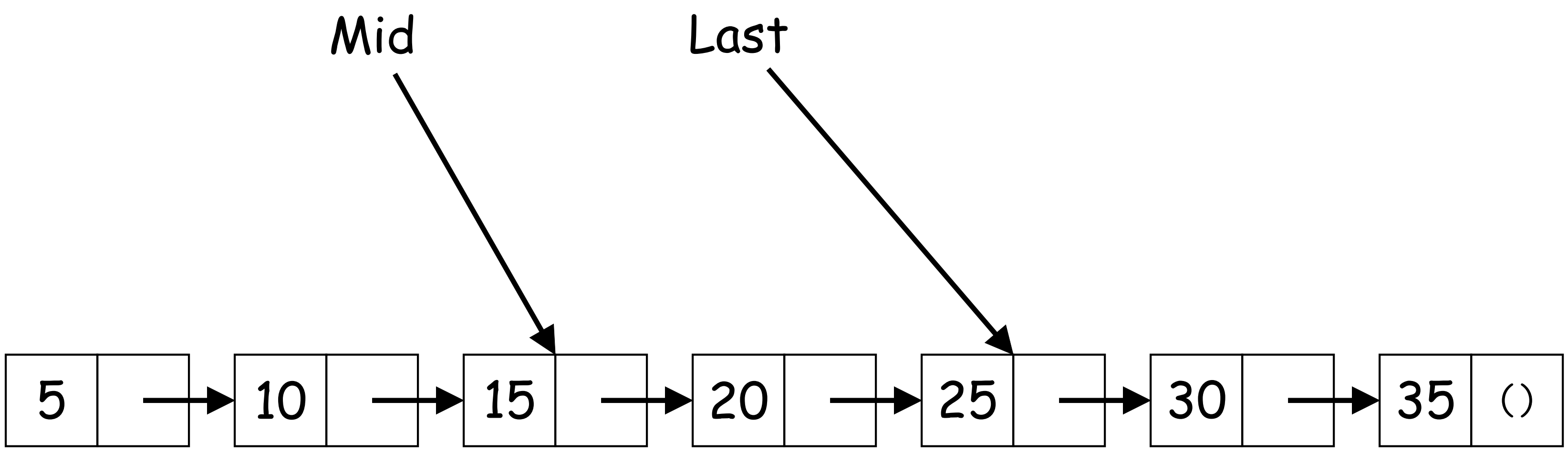

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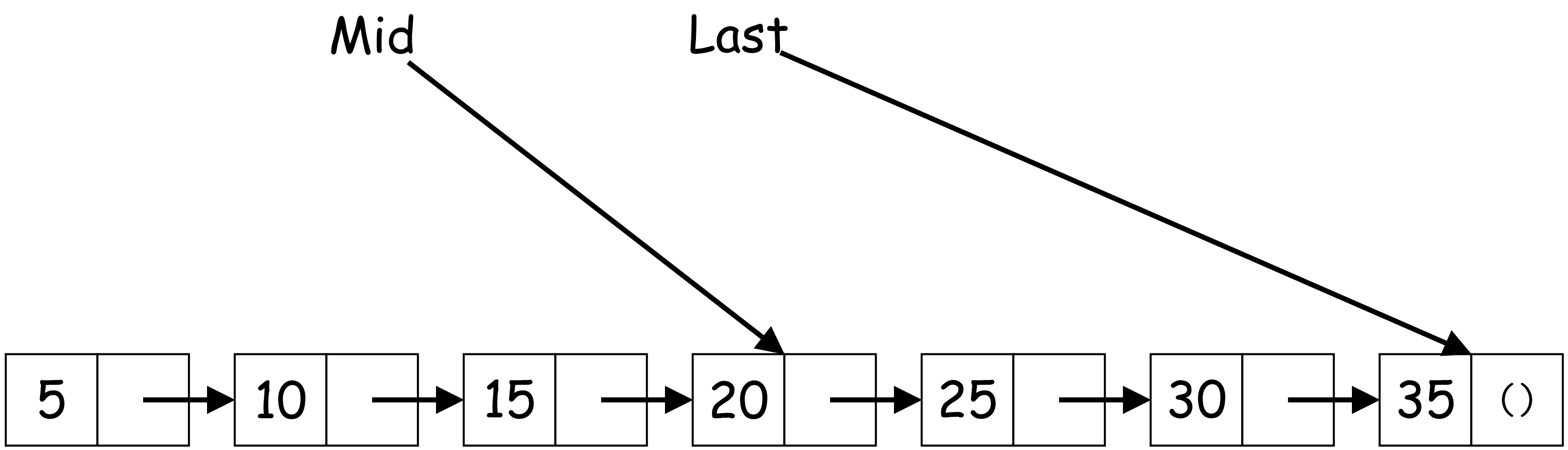

Exercise: Intersperse Lists (I)

We seek a recursive solution to the following:

```
def intersperse(L, pred, inserts):
    """Returns a copy of linked list L in which the items whose
    values satisfy PRED (a one-argument, boolean function) are
    followed by successive values from linked list INSERTS, until
    INSERTS is exhausted. The function is non-destructive.
    >>> data = toLinked([1, 2, 3, 4, 5])
    >>> alt = toLinked([10, 11, 12, 13])
    >>> print(intersperse(data, lambda x: x % 2 == 1, alt))
    <1 10 2 3 11 4 5 12>
    >>> print(intersperse(data, lambda x: True, alt))
    <1 10 2 11 3 12 4 13 5>
    """
```

Exercise: Intersperse Lists (II)

This time, give an iterative solution. Here, we'll use a dummy *sentinel node* just before the beginning of the resulting list to cut down on special cases.

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def intersperse2(L, pred, inserts):
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    sentinel = Link(None)
    # Code
    return sentinel.rest

intersperse2(toList(1, 2, 3, 4, 5), lambda x: x%2 == 1, toList(10, 11))
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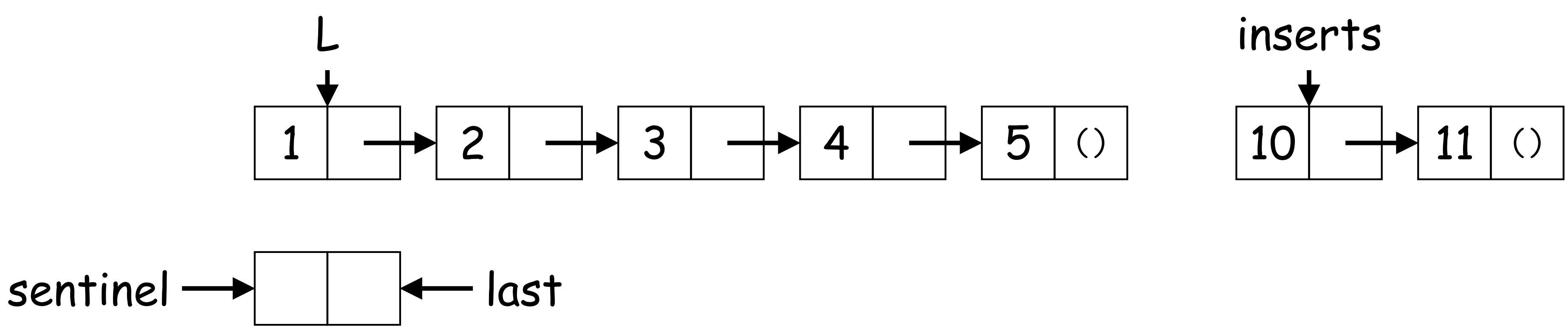

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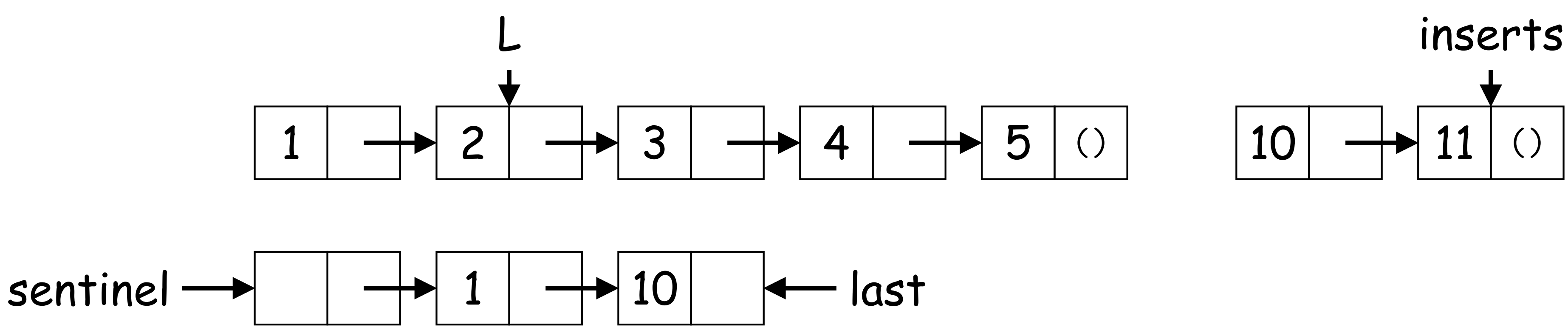

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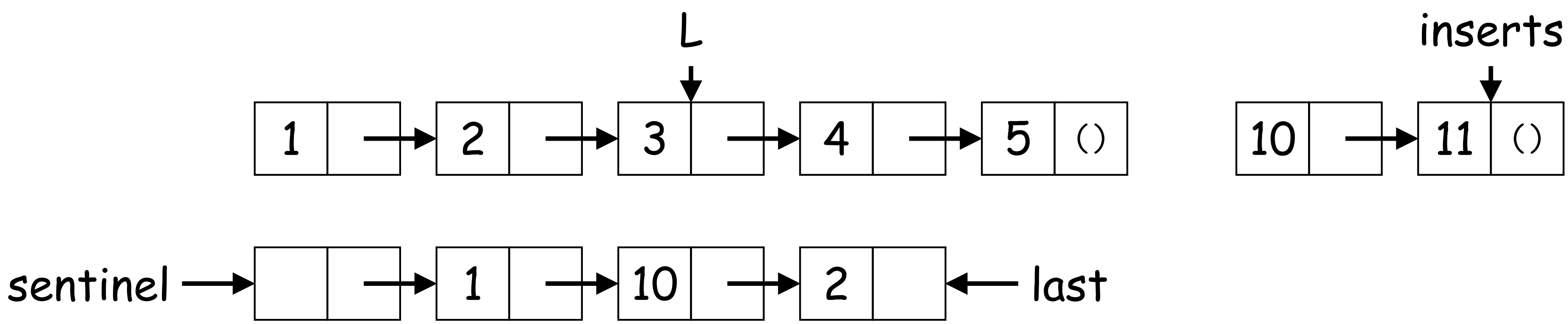

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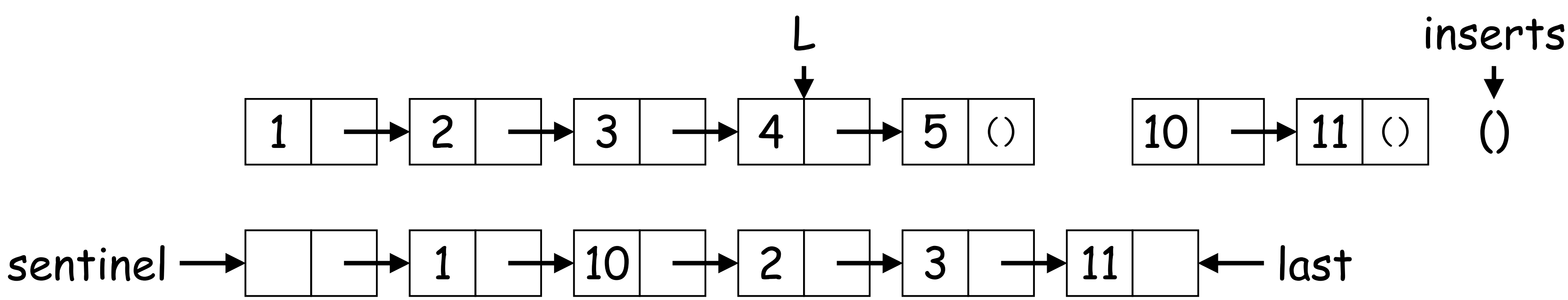

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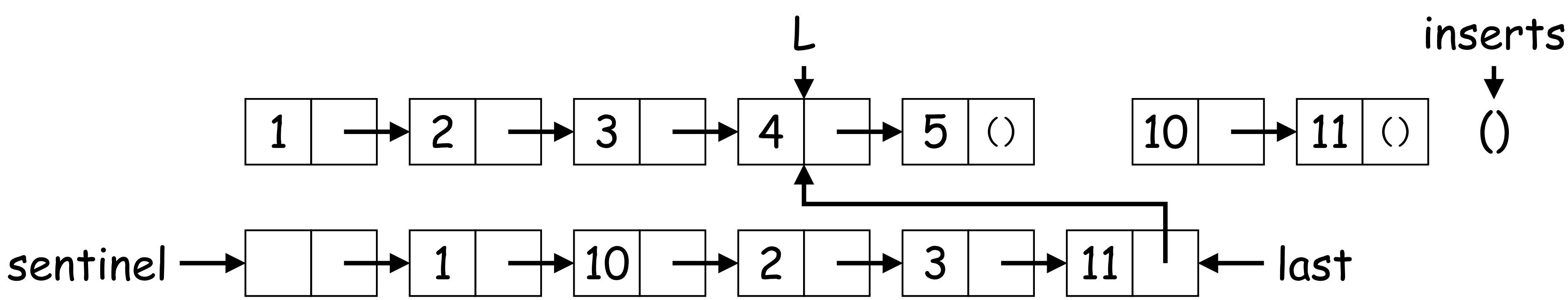

Exercise: Intersperse Lists (III)

This time, give a recursive, *destructive* solution. That is, we do not create any new Links, but instead modify existing ones as needed to create the result list, possibly destroying the original data in the `L` and `inserts` arguments.

```
def dintersperse(L, pred, inserts):
    """Returns a copy of linked list L in which the items whose
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```

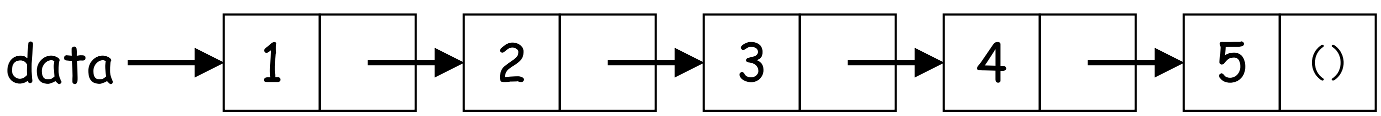

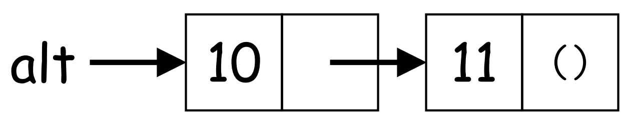

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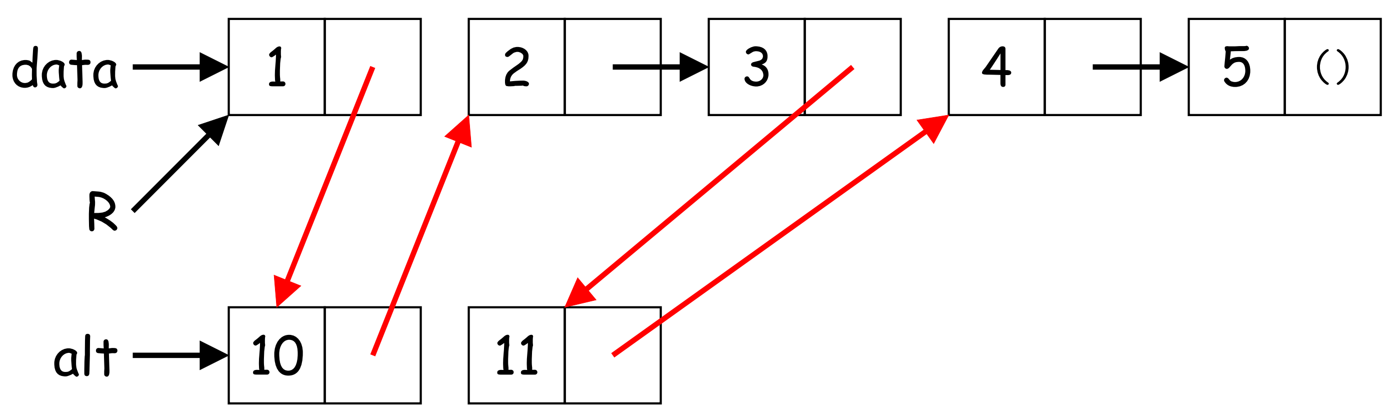


Binary Trees

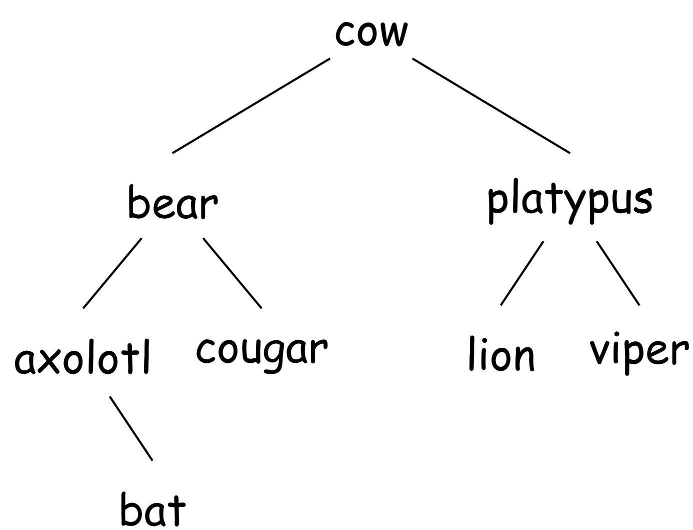
- We've looked at trees in lecture and homework.
- Let's consider a slight variation of what you've seen that is specialized to binary trees:

```
class BTree(Tree):
    """A tree with exactly two branches, which may be empty."""
    empty = None   # Placeholder

    def __init__(self, label, left=None, right=None):
        self.label = label
        self.left = left or BTree.empty
        self.right = right or BTree.empty

    def __repr__(self):
        return ...

BTree.empty = BTree(None)
```

- As it happens, I could simply have left `BTree.empty` as `None`. However, I wanted to make it a special (unique) `BTree` node. What might this be useful for, and why did I have to do it this way?

Binary Search Trees

- We just saw binary search trees on Wednesday in the context of Scheme.
- Of course, the same algorithms apply to Python.
- Today, we'll use simple strings as labels. All labels in a left subtree of T must be lexicographically less than T's label, and all labels in the right subtree must be greater.



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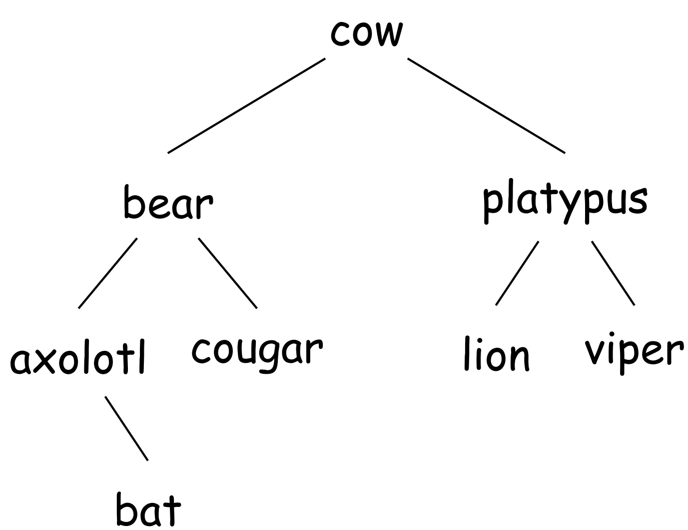


```
def isIn(T, target):
    """Return True iff T contains the label TARGET."""
    ???
```

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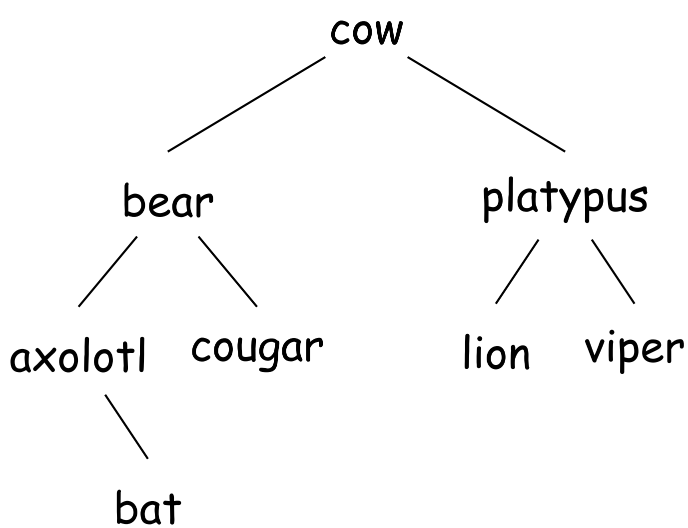


```
def add(T, target):
    """Destructively modify T to add the label TARGET, if it is not already
    present.  Return the resulting tree."""
    ???
```