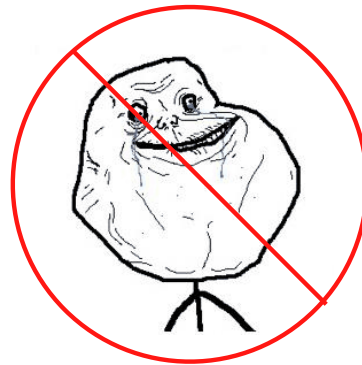


Higher-Order Functions

Announcements

Office Hours: You Should Go!

You are not alone!



<https://cs61a.org/office-hours/>

Example: Prime Factorization

Prime Factorization

Each positive integer n has a set of prime factors: primes whose product is n

...

$$8 = 2 * 2 * 2$$

$$9 = 3 * 3$$

$$10 = 2 * 5$$

$$11 = 11$$

$$12 = 2 * 2 * 3$$

...

One approach: Find the smallest prime factor of n , then divide by it

$$858 = 2 * 429 = 2 * 3 * 143 = 2 * 3 * 11 * 13$$

(Demo)

Designing Functions

Describing Functions

A function's *domain* is the set of all inputs it might possibly take as arguments.

A function's *range* is the set of output values it might possibly return.

A pure function's *behavior* is the relationship it creates between input and output.

```
def square(x):  
    """Return X * X."""
```

x is a number

square returns a non-negative real number

square returns the square of x

A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)      >>> round(1.23, 1)    >>> round(1.23, 0)    >>> round(1.23, 5)
1                    1.2                    1                    1.23
```

Don't repeat yourself (DRY): Implement a process just once, but execute it many times

(Demo)

Higher-Order Functions

Summation Example

```
def cube(k):  
    return pow(k, 3)
```

Function of a single argument
(*not called "term"*)

```
def summation(n, term):  
    """Sum the first n terms of a sequence.
```

A formal parameter that will
be bound to a function

```
>>> summation(5, cube)
```

```
225
```

The cube function is passed
as an argument value

```
    """  
    total, k = 0, 1  
    while k <= n:  
        total, k = total + term(k), k + 1  
    return total
```

0 + 1 + 8 + 27 + 64 + 125

The function bound to term
gets called here

Twenty-One Rules

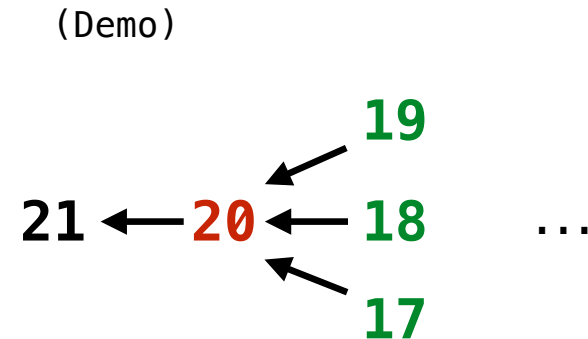
Two players alternate turns, on which they can add 1, 2, or 3 to the current total

The total starts at 0

The game end whenever the total is 21 or more

The last player to add to the total loses

Some states are good; some are bad



(Demo)

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that returns a function

```
def make_adder(n):  
    """Return a function that takes one argument k and returns k + n.  
  
    >>> add_three = make_adder(3)  
    >>> add_three(4)  
    7  
    """  
    def adder(k):  
        return k + n  
    return adder
```

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function