61A Extra Lecture 1
Announcements

- If you want 1 unit (pass/no pass) of credit for in CS 98-52, the CCN is 34591.
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• All info and materials will be posted to cs61a.org/extra.html
Newton's Method
Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!
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\[ f(x) = x^2 - 2 \]
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A "zero" of a function $f$ is an input $x$ such that $f(x) = 0$.

$f(x) = x^2 - 2$
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$$f(x) = x^2 - 2$$

$x=1.414213562373095$
Newton's Method Background

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Application: a method for computing square roots, cube roots, etc.
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A "zero" of a function $f$ is an input $x$ such that $f(x)=0$.

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)
Newton's Method

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Compute the value of $f$ at the guess: $f(x)$
Newton's Method

Given a function $f$ and initial guess $x$,

Repeatedly improve $x$:

- Compute the value of $f$ at the guess: $f(x)$
- Compute the derivative of $f$ at the guess: $f'(x)$
Newton's Method

Given a function f and initial guess x,

Repeatedly improve x:

Compute the value of f at the guess: f(x)

Compute the derivative of f at the guess: f'(x)

Update guess x to be:

\[ x = \frac{f(x)}{f'(x)} \]
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Update guess $x$ to be:

$$x - \frac{f(x)}{f'(x)}$$
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$$x \rightarrow x - \frac{f(x)}{f'(x)}$$
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Finish when $f(x) = 0$ (or close enough)
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Finish when $f(x) = 0$ (or close enough)
Using Newton's Method
Using Newton's Method

How to find the square root of 2?
Using Newton's Method

How to find the square root of 2?

```python
>>> f = lambda x: x**2 - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```
Using Newton's Method

How to find the square root of 2?

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\[ f(x) = x^2 - 2 \]
\[ f'(x) = 2x \]
Using Newton's Method

How to find the square root of 2?

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\[ f'(x) = 2x \]

\[
\begin{align*}
\text{Appplies Newton's method} \\
\text{find_zero}(f, df) \\
1.4142135623730951
\end{align*}
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Using Newton's Method

How to find the square root of 2?

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\[ f'(x) = 2x \]

\[
\begin{align*}
\text{>>> } f &= \text{lambda } x: x**2 - 2 \\
\text{>>> } df &= \text{lambda } x: 2x \\
\text{>>> } \text{find_zero}(f, df) \\
1.4142135623730951
\end{align*}
\]

How to find the cube root of 729?

Applies Newton's method
Using Newton's Method

How to find the square root of 2?

```python
>>> f = lambda x: x**2 - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

How to find the cube root of 729?

```python
3
V
```
Using Newton's Method

How to find the square root of 2?

\[ f(x) = x^2 - 2 \]
\[ f'(x) = 2x \]

```python
>>> f = lambda x: x**2 - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

How to find the cube root of 729?

\[ g(x) = x^3 - 729 \]
\[ g'(x) = 3x^2 \]

```python
>>> g = lambda x: x**3 - 729
>>> dg = lambda x: 3*x**2
>>> find_zero(g, dg)
9.0
```
Using Newton's Method

How to find the square root of 2?

$$f(x) = x^2 - 2$$
$$f'(x) = 2x$$

>>> f = lambda x: x**2 - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951

How to find the cube root of 729?

$$g(x) = x^3 - 729$$
$$g'(x) = 3x^2$$

>>> g = lambda x: x**3 - 729
>>> dg = lambda x: 3*x**2
>>> find_zero(g, dg)
9.0
Iterative Improvement
Special Case: Square Roots
Special Case: Square Roots

How to compute square_root(a)

**Idea:** Iteratively refine a guess $x$ about the square root of $a$
Special Case: Square Roots

How to compute $\text{square\_root}(a)$

**Idea:** Iteratively refine a guess $x$ about the square root of $a$

**Update:**
Special Case: Square Roots

How to compute square_root(a)

**Idea:** Iteratively refine a guess $x$ about the square root of $a$

$$x = \frac{x + \frac{a}{x}}{2}$$
Special Case: Square Roots

How to compute square_root(a)

**Idea:** Iteratively refine a guess $x$ about the square root of $a$

**Update:** $x = \frac{x + \frac{a}{x}}{2}$

Babylonian Method
Special Case: Square Roots

How to compute $\text{square\_root}(a)$

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**Update:**

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(Babylonian Method)
Special Case: Square Roots

How to compute \( \text{square\_root}(a) \)

**Idea:** Iteratively refine a guess \( x \) about the square root of \( a \)

\[
\text{Update:} \quad x = \frac{x + \frac{a}{x}}{2}
\]

Implementation questions:
**Special Case: Square Roots**

How to compute square_root(a)

**Idea:** Iteratively refine a guess $x$ about the square root of $a$

\[
X = \frac{X + \frac{a}{X}}{2}
\]

**Implementation questions:**

What guess should start the computation?
Special Case: Square Roots

How to compute square_root(a)

**Idea:** Iteratively refine a guess $x$ about the square root of $a$

$\text{Update: } x = \frac{x + \frac{a}{x}}{2}$

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?
Special Case: Cube Roots
Special Case: Cube Roots

How to compute $\text{cube_root}(a)$

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$
Special Case: Cube Roots

How to compute cube_root(a)

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$

**Update:**
Special Case: Cube Roots

How to compute $\text{cube}_\text{root}(a)$

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$

**Update:**

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$
**Special Case: Cube Roots**

How to compute $\text{cube_root}(a)$

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$

**Update:**

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$  \hspace{1cm} \text{(Demo)}$$
Special Case: Cube Roots

How to compute \texttt{cube_root}(a)

\textbf{Idea:} Iteratively refine a guess \( x \) about the cube root of \( a \)

**Update:** \[ x = \frac{2 \cdot x + \frac{a}{x^2}}{3} \] (Demo)

\textbf{Implementation questions:}
Special Case: Cube Roots

How to compute $\text{cube\_root}(a)$

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$

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What guess should start the computation?
Special Case: Cube Roots

How to compute $\text{cube\_root}(a)$

**Idea:** Iteratively refine a guess $x$ about the cube root of $a$

**Update:**

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

(Demo)

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?
Implementing Newton's Method

(Demo)
Extensions
Approximate Differentiation
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Differentiation can be performed symbolically or numerically

![Graph showing approximate differentiation](image-url)
Approximate Differentiation

Differentiation can be performed symbolically or numerically

\[ f(x) = x^2 - 16 \]
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Approximate Differentiation

Differentiation can be performed symbolically or numerically

\[ f(x) = x^2 - 16 \]
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\[ f'(x) = \lim_{a \to 0} \frac{f(x + a) - f(x)}{a} \]
Approximate Differentiation

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\[ f(x) = x^2 - 16 \]
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\[ f'(2) = 4 \]

\[ f'(x) = \lim_{a \to 0} \frac{f(x + a) - f(x)}{a} \]
\[ f'(x) \approx \frac{f(x + a) - f(x)}{a} \]
Approximate Differentiation

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\[ f'(x) \approx \frac{f(x + a) - f(x)}{a} \quad \text{(if } a \text{ is small)} \]
Approximate Differentiation

Differentiation can be performed symbolically or numerically

\[ f(x) = x^2 - 16 \]
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\begin{align*}
f'(x) &= \lim_{a \to 0} \frac{f(x + a) - f(x)}{a} \\
f'(x) &\approx \frac{f(x + a) - f(x)}{a} \quad \text{(if } a \text{ is small)}
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(Demo)
Critical Points and Inverses
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Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0.
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(Demo)
Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0

(Demo)

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value $x$ such that $f(x) = y$
Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0.

\[(\text{Demo})\]

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value $x$ such that $f(x) = y$.

\[(\text{Demo})\]

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg