61A Extra Lecture 1

Announcements

• If you want 1 unit (pass/no pass) of credit for this CS 98, stay tuned for a Piazza post
  • Only for people who really want extra work that’s beyond the scope of normal CS 61A
  • Anyone is welcome to attend the extra lectures, whether or not they enroll
  • Permanent lecture times are TBD, but probably Wednesday evening or Monday evening

Newton’s Method

Newton’s Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!

Application: a method for computing square roots, cube roots, etc.

The positive zero of \( f(x) = x^2 - a \) is \( \sqrt{a} \). (We’re solving the equation \( x^2 = a \).)

Using Newton’s Method

\[
\text{How to find the square root of 2?}
\]

\[
\text{How to find the cube root of 729?}
\]

Special Case: Square Roots

How to compute square_root(a)?

Idea: Iteratively refine a guess \( x \) about the square root of \( a \).

Update: \( x = \frac{x + \frac{a}{x}}{2} \) (Iterates)

Implementation questions:

What guess should start the computation?

How do we know when we are finished?
Special Case: Cube Roots

How to compute \( \text{cube_root}(a) \)

Idea: Iteratively refine a guess \( x \) about the cube root of \( a \)

\[
\text{Update: } x \leftarrow \frac{2}{3} x + \frac{a}{3x^2}
\]

Implementation questions:
- What guess should start the computation?
- How do we know when we are finished?

Implementing Newton's Method

Approximate Differentiation

Differentiation can be performed symbolically or numerically

\[
f(x) = x^2 - 16
\]

\[
f'(x) = 2x
\]

\[
f'(2) = 4
\]

\[
f'(x) \approx \frac{f(x+h) - f(x)}{h} \quad \text{(if } h \text{ is small)}
\]

Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0

The inverse \( f^{-1}(y) \) of a differentiable, one-to-one function computes the value \( x \) such that \( f(x) = y \)