1 More Recursion

Questions

1.1 In discussion 1, we implemented the function is_prime, which takes in a positive integer and returns whether or not that integer is prime, iteratively.

Now, let’s implement it recursively! As a reminder, an integer is considered prime if it has exactly two unique factors: 1 and itself.

```python
def is_prime(n):
    """
    >>> is_prime(7)
    True
    >>> is_prime(10)
    False
    >>> is_prime(1)
    False
    """
    def prime_helper(____________________):
        if __________________________:
            __________________________
        elif __________________________:
            __________________________
        else:
            __________________________
        return __________________________
```
1.2 Define a function `make_fn_repeater` which takes in a one-argument function \( f \) and an integer \( x \). It should return another function which takes in one argument, another integer. This function returns the result of applying \( f \) to \( x \) this number of times.

Make sure to use recursion in your solution.

```python
def make_func_repeater(f, x):
    
    >>> incr_1 = make_func_repeater(lambda x: x + 1, 1)
    >>> incr_1(2)  #same as f(f(x))
    3
    >>> incr_1(5)
    6
    
    def repeat(______________):
        if ________________:
            return ________________
        else:
            return ________________
        return ________________
```

2 Tree Recursion

Consider a function that requires more than one recursive call. A simple example is the recursive fibonacci function:

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

This type of recursion is called tree recursion, because it makes more than one recursive call in its recursive case. If we draw out the recursive calls, we see the recursive calls in the shape of an upside-down tree:

```
fib(4)
   / \
fib(3)  fib(2)
  /   /   /
fib(2) fib(1) fib(1) fib(0)
```

We could, in theory, use loops to write the same procedure. However, problems that are naturally solved using tree recursive procedures are generally difficult to write iteratively. It is sometimes the case that a tree recursive problem also involves iteration: for example, you might use a while loop to add together multiple recursive calls.

As a general rule of thumb, whenever you need to try multiple possibilities at the same time, you should consider using tree recursion.

Questions

2.1 I want to go up a flight of stairs that has $n$ steps. I can either take 1 or 2 steps each time. How many different ways can I go up this flight of stairs? Write a function `count_stair_ways` that solves this problem for me. Assume $n$ is positive.
Before we start, what’s the base case for this question? What is the simplest input?

What do `count_stair_ways(n - 1)` and `count_stair_ways(n - 2)` represent?

Use those two recursive calls to write the recursive case:

```python
def count_stair_ways(n):
```

2.2 Consider a special version of the `count_stairways` problem, where instead of taking 1 or 2 steps, we are able to take **up to and including** \( k \) steps at a time.

Write a function `count_k` that figures out the number of paths for this scenario. Assume \( n \) and \( k \) are positive.

```python
def count_k(n, k):
    """
    >>> count_k(3, 3) # 3, 2 + 1, 1 + 2, 1 + 1 + 1
    4
    >>> count_k(4, 4)
    8
    >>> count_k(10, 3)
    274
    >>> count_k(300, 1) # Only one step at a time
    1
    """
```
Here's a part of the Pascal's triangle:

<table>
<thead>
<tr>
<th>Column: 0 1 2 3 4 ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 0: 1</td>
</tr>
<tr>
<td>Row 1: 1 1</td>
</tr>
<tr>
<td>Row 2: 1 2 1</td>
</tr>
<tr>
<td>Row 3: 1 3 3 1</td>
</tr>
<tr>
<td>Row 4: 1 4 6 4 1</td>
</tr>
<tr>
<td>...</td>
</tr>
</tbody>
</table>

Every number in Pascal's triangle is defined as the sum of the item above it and the item that is directly to the upper left of it, use 0 if the entry is empty. Define the procedure `pascal(row, column)` which takes a row and a column, and finds the value at that position in the triangle.

```python
def pascal(row, column):
```