CS 61A Fall 2018

Recursion & Tree Recursion

Discussion 3: September 12, 2018 Solutions

1 More Recursion

Questions

1.1 In discussion 1, we implemented the function `is_prime`, which takes in a positive integer and returns whether or not that integer is prime, iteratively.

Now, let's implement it recursively! As a reminder, an integer is considered prime if it has exactly two unique factors: 1 and itself.

```
def is_prime(n):
    """
    >>> is_prime(7)
    True
    >>> is_prime(10)
    False
    >>> is_prime(1)
    False
    """
    def prime_helper(____________________):

        if   ________________________:

             ________________________

        elif ________________________:

             ________________________

        else:

             ________________________

    return __________________________
```

```
def prime_helper(index):
    if index == n:
        return True
    elif n % index == 0 or n == 1:
        return False
    else:
        return prime_helper(index + 1)
return prime_helper(2)
```

1.2 Define a function `make_fn_repeater` which takes in a one-argument function `f` and an integer `x`. It should return another function which takes in one argument, another integer. This function returns the result of applying `f` to `x` this number of times.

Make sure to use recursion in your solution.

```
def make_func_repeater(f, x):
    """
    >>> incr_1 = make_func_repeater(lambda x: x + 1, 1)
    >>> incr_1(2) #same as f(f(x))
    3
    >>> incr_1(5)
    6
    """

    def repeat(___________________):

        if _______________________:

            return __________________

        else:

            return __________________

    return _________________________
```

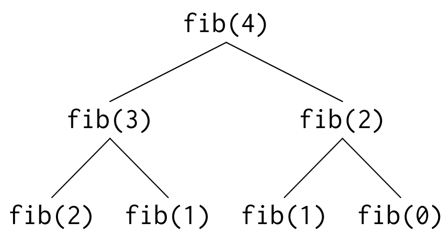
```
    def repeat(i):
        if i == 0:
            return x
        else:
            return f(repeat(i - 1))
    return repeat
```

2 Tree Recursion

Consider a function that requires more than one recursive call. A simple example is the recursive `fibonacci` function:

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

This type of recursion is called `tree recursion`, because it makes more than one recursive call in its recursive case. If we draw out the recursive calls, we see the recursive calls in the shape of an upside-down tree:



We could, in theory, use loops to write the same procedure. However, problems that are naturally solved using tree recursive procedures are generally difficult to write iteratively. It is sometimes the case that a tree recursive problem also involves iteration: for example, you might use a while loop to add together multiple recursive calls.
As a general rule of thumb, whenever you need to try multiple possibilities at the same time, you should consider using tree recursion.

How to diagram Tree Recursion

Questions

2.1 I want to go up a flight of stairs that has `n` steps. I can either take 1 or 2 steps each time. How many different ways can I go up this flight of stairs? Write a function `count_stair_ways` that solves this problem for me. Assume `n` is positive.

Before we start, what's the base case for this question? What is the simplest input?

When there is only 1 step, there is only one way to go up the stair. When there are two steps, we can go up in two ways: take a two-step, or take 2 one-steps.

What do `count_stair_ways(n - 1)` and `count_stair_ways(n - 2)` represent?

`count_stair_ways(n - 1)` represents the number of different ways to go up the first $n - 1$ stairs. `count_stair_ways(n - 2)` represents the number of different ways to go up the first $n - 2$ stairs. Our base cases will take care of the remaining 1 or 2 steps.

Use those two recursive calls to write the recursive case:

```
def count_stair_ways(n):


    if n == 1:
        return 1
    elif n == 2:
        return 2
    return count_stair_ways(n-1) + count_stair_ways(n-2)
```

Video walkthrough (Leap of Faith)

Video Walkthrough (Diagramming Trees)

2.2 Consider a special version of the `count_stairways` problem, where instead of taking 1 or 2 steps, we are able to take **up to and including** `k` steps at a time.

Write a function `count_k` that figures out the number of paths for this scenario. Assume `n` and `k` are positive.

```
def count_k(n, k):
    """
    >>> count_k(3, 3) # 3, 2 + 1, 1 + 2, 1 + 1 + 1
    4
    >>> count_k(4, 4)
    8
    >>> count_k(10, 3)
    274
    >>> count_k(300, 1) # Only one step at a time
    1
    """


    if n == 0:
        return 1
    elif n < 0:
        return 0
    else:
        total = 0
```

```
    i = 1
    while i <= k:
        total += count_k(n - i, k)
        i += 1
    return total
```

Video Walkthrough

2.3 Here's a part of the Pascal's triangle:

```
Column:  0    1    2    3    4    ...
Row 0:   1
Row 1:   1    1
Row 2:   1    2    1
Row 3:   1    3    3    1
Row 4:   1    4    6    4    1
...
```

Every number in Pascal's triangle is defined as the sum of the item above it and the item that is directly to the upper left of it, use `0` if the entry is empty. Define the procedure `pascal(row, column)` which takes a row and a column, and finds the value at that position in the triangle.

```
def pascal(row, column):


    if column == 0:
        return 1
    elif row == 0:
        return 0
    else:
        return pascal(row - 1, column) + \
               pascal(row - 1, column - 1)
```

Background: Pascal's triangle is a useful recursive definition that tells us the coefficients in the expansion of the polynomial $(x + a)^n$. Each element in the triangle has a coordinate, given by the row it is on and its position in the row (which you could call its column).