1 Nonlocal

Until now, you've been able to access names in parent frames, but you have not been able to modify them. The `nonlocal` keyword can be used to modify a binding in a parent frame. For example, consider `stepper`, which uses `nonlocal` to modify `num`:

```python
def stepper(num):
    def step():
        nonlocal num # declares num as a nonlocal name
        num = num + 1 # modifies num in the stepper frame
        return num
    return step
```

```bash
>>> step1 = stepper(10)
>>> step1() # Modifies and returns num
11
>>> step1() # num is maintained across separate calls to step
12
>>> step2 = stepper(10) # Each returned step function keeps its own state
>>> step2()
11
```

As illustrated in this example, `nonlocal` is useful for maintaining state across different calls to the same function.

However, there are two important caveats with `nonlocal` names:

- **Global names** cannot be modified using the `nonlocal` keyword.

- **Names in the current frame** cannot be overridden using the `nonlocal` keyword. This means we cannot have both a local and nonlocal binding with the same name in a single frame.

Because `nonlocal` lets you modify bindings in parent frames, we call functions that use it **mutable functions**.
Questions

1.1 Draw the environment diagram for the following code.

```python
def stepper(num):
    def step():
        nonlocal num
        num = num + 1
        return num
    return step

s = stepper(3)
s()
s()
```

Video walkthrough
Write a function that takes in a number $n$ and returns a one-argument function. The returned function takes in a function that is used to update $n$. It should return the updated $n$.

```python
def memory(n):
    ""
    >>> f = memory(10)
    >>> f(lambda x: x * 2)
    20
    >>> f(lambda x: x - 7)
    13
    >>> f(lambda x: x > 5)
    True
    ""

def f(g):
    nonlocal n
    n = g(n)
    return n
    return f
```

Video walkthrough
Write a function that takes in no arguments and returns two functions, `prepend` and `get`, which represent the “add to front of list” and “get the ith item” operations, respectively. Do not use any python built-in data structures like lists or dictionaries. You do not necessarily need to use all the lines.

This question is more difficult than the average discussion problem; it is an exam level problem.

```python
def nonlocalist():
    """
    >>> prepend, get = nonlocalist()
    >>> prepend(2)
    >>> prepend(3)
    >>> prepend(4)
    >>> get(0)
    4
    >>> get(1)
    3
    >>> get(2)
    2
    >>> prepend(8)
    >>> get(2)
    3
    """
    get = lambda x: "Index out of range!"
    def prepend(value):
        f = ________________________________
        def get(i):
            if i == 0:
                return value
            ________________________________
        return __________________________, _________________________
    get = lambda x: "Index out of range!"
    def prepend(value):
        nonlocal get
        f = get
        def get(i):
            if i == 0:
                return value
            return f(i - 1)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion session.
return prepend, lambda x: get(x)
2 Iterators

An iterable is a data type which contains a collection of values which can be processed one by one sequentially. Some examples of iterables we’ve seen include lists, tuples, strings, and dictionaries. In general, any object that can be iterated over in a for loop can be considered an iterable.

While an iterable contains values that can be iterated over, we need another type of object called an iterator to actually retrieve values contained in an iterable. Calling the iter function on an iterable will create an iterator over that iterable. Each iterator keeps track of its position within the iterable. Calling the next function on an iterator will give the current value in the iterable and move the iterator’s position to the next value.

In this way, the relationship between an iterable and an iterator is analogous to the relationship between a book and a bookmark - an iterable contains the data that is being iterated over, and an iterator keeps track of your position within that data.

Once an iterator has returned all the values in an iterable, subsequent calls to next on that iterable will result in a StopIteration exception. In order to be able to access the values in the iterable a second time, you would have to create a second iterator. One important application of iterables and iterators is the for loop. We’ve seen how we can use for loops to iterate over iterables like lists and dictionaries.

This only works because the for loop implicitly creates an iterator using the built-in function. Python then calls next repeatedly on the iterator, until it raises StopIteration.

The code to the right shows how we can mimic the behavior of for loops using while loops.

Note that most iterators are also iterables - that is, calling iter on them will return an iterator. This means that we can use them inside for loops. However, calling iter on most iterators will not create a new iterator - instead, it will simply return the same iterator.

We can also iterate over iterables in a list comprehension or pass in an iterable to the built-in function list in order to put the items of an iterable into a list.

In addition to the sequences we’ve learned, Python has some built-in ways to create iterables and iterators. Here are a few useful ones:

- range(start, end) returns an iterable containing numbers from start to end. If start is not provided, it defaults to 0.
- map(f, iterable) returns a new iterator containing the values resulting from applying f to each value in iterable.
- filter(f, iterable) returns a new iterator containing only the values in iterable for which f(value) returns True.

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
Questions

2.1 What would Python display? If a StopIteration Exception occurs, write StopIteration, and if another error occurs, write Error.

```python
>>> lst = [6, 1, "a"]
>>> next(lst)

Error

>>> lst_iter = iter(lst)
>>> next(lst_iter)

6

>>> next(lst_iter)

1

>>> next(iter(lst))

6

>>> [x for x in lst_iter]

['a']
```
Generators

A generator function is a special kind of Python function that uses a `yield` statement instead of a `return` statement to report values. When a generator function is called, it returns a generator object, which is a type of iterator. To the right, you can see a function that returns an iterator over the natural numbers.

The `yield` statement is similar to a `return` statement. However, while a `return` statement closes the current frame after the function exits, a `yield` statement causes the frame to be saved until the next time `next` is called, which allows the generator to automatically keep track of the iteration state.

Once `next` is called again, execution resumes where it last stopped and continues until the next `yield` statement or the end of the function. A generator function can have multiple `yield` statements.

Including a `yield` statement in a function automatically tells Python that this function will create a generator. When we call the function, it returns a generator object instead of executing the body. When the generator's `next` method is called, the body is executed until the next `yield` statement is executed.

When `yield from` is called on an iterator, it will `yield` every value from that iterator. It’s similar to doing the following:

```python
for x in an_iterator:
    yield x
```

The example to the right demonstrates different ways of computing the same result.

```
>>> def gen_naturals():
...     current = 0
...     while True:
...         yield current
...         current += 1
... >>> gen = gen_naturals()
>>> gen
<generator object gen at ...>
>>> next(gen)
0
>>> next(gen)
1
```
Questions

2.1 Write a generator function `merge` that takes in two infinite generators `a` and `b` that are in increasing order without duplicates and returns a generator that has all the elements of both generators, in increasing order, without duplicates.

```python
def merge(a, b):
    """
    >>> def sequence(start, step):
    ...     while True:
    ...         yield start
    ...     start += step
    >>> a = sequence(2, 3) # 2, 5, 8, 11, 14, ...
    >>> b = sequence(3, 2) # 3, 5, 7, 9, 11, 13, 15, ...
    >>> result = merge(a, b) # 2, 3, 5, 7, 8, 9, 11, 13, 14, 15
    >>> [next(result) for _ in range(10)]
    [2, 3, 5, 7, 8, 9, 11, 13, 14, 15]
    """

    first_a, first_b = next(a), next(b)
    while True:
        if first_a == first_b:
            yield first_a
            first_a, first_b = next(a), next(b)
        elif first_a < first_b:
            yield first_a
            first_a = next(a)
        else:
            yield first_b
            first_b = next(b)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
2.2 Write a generator function `generate_subsets` that returns all subsets of the positive integers from 1 to \( n \). Each call to this generator’s `next` method will return a list of subsets of the set \([1, 2, \ldots, n]\), where \( n \) is the number of previous calls to `next`.

```python
def generate_subsets():
    ""
    >>> subsets = generate_subsets()
    >>> for _ in range(3):
    ...     print(next(subsets))
    ...
    []
    [[]]
    [[], [1]]
    [[], [1], [2], [1, 2]]
    ""

    subsets = [[]]
    n = 1
    while True:
        yield subsets
        subsets = subsets + [s + [n] for s in subsets]
        n += 1
```

We start with a base list of subsets. To get the next sequence of subsets, we need two things:

- All current subsets will continue to be valid subsets in the future.
- We take all the subsets we currently have, and add the next number. These are also valid subsets.
2.3 Implement `sum_paths_gen`, which takes in a tree `t` and returns a generator which yields the sum of all the nodes from a path from the root of a tree to a leaf.

You may yield the sums in any order.

```python
def sum_paths_gen(t):
    ""
    >>> t1 = tree(5)
    >>> next(sum_paths_gen(t1))
    5
    >>> t2 = tree(1, [tree(2, [tree(3), tree(4)]), tree(9)])
    >>> sorted(sum_paths_gen(t2))
    [6, 7, 10]
    ""
    if ________________:
        yield _______________
    for ________________:
        for ________________:
            yield _______________
```

```python
def sum_paths_gen(t):
    if is_leaf(t):
        yield label(t)
    for b in branches(t):
        for s in sum_paths_gen(b):
            yield s + label(t)
```
1. Trie Recursion

A trie is a type of tree where the values of each node are letters representing part of a larger word. A valid word is a string containing the letters along any path from root to leaf. For simplicity, assume that our trie is represented with the tree abstract data type and where the value of each node contains just a single letter.

*Recall:* The tree abstract data type is defined with the following constructors and selectors.

```python
def tree(label, branches=[]):
    """Construct a tree with the given label value and a list of branches."""

def label(tree):
    """Return the label value of a tree."""

def branches(tree):
    """Return the list of branches of the given tree."""

def is_tree(tree):
    """Returns True if the given tree is a tree, and False otherwise."""

def is_leaf(tree):
    """Returns True if the given tree's list of branches is empty, and False otherwise."""
```

Implement `collect_words`, which takes in a trie `t` and returns a Python list containing all the words contained in the trie.

```python
>>> greetings = tree('h', [tree('i'),
    ... tree('e', [tree('l', [tree('l', [tree('o')])]),
    ... tree('y')])])
>>> print_tree(greetings)
h
  i
  e
    l
      o
    y
def collect_words(t):
    """Return a list of all the words contained in the tree where the value of each node in the tree is an individual letter. Words terminate at the leaf of a tree.

    >>> collect_words(greetings)
    ['hi', 'hello', 'hey']"""

    if is_leaf(t):
        return [label(t)]
    words = []
    for branch in branches(t):
        words += [label(t) + word for word in collect_words(branch)]
    return words
```