1 Interfaces

In computer science, an interface is a shared set of attributes, along with a specification of the attributes’ behavior. For example, an interface for vehicles might consist of the following methods:

- `def drive(self):` Drives the vehicle if it has stopped.
- `def stop(self):` Stops the vehicle if it is driving.

Data types can implement the same interface in different ways. For example, a `Car` class and a `Train` can both implement the interface described above, but the `Car` probably has a different mechanism for `drive` than the `Train`.

The power of interfaces is that other programs don’t have to know how each data type implements the interface – only that they have implemented the interface. The following `travel` function can work with both `Cars` and `Trains`:

```python
def travel(vehicle):
    while not at_destination():
        vehicle.drive()
        vehicle.stop()
```

Magic Methods

Python defines many interfaces that can be implemented by user-defined classes. For example, the interface for arithmetic consists of the following methods:

- `def __add__(self, other):` Allows objects to do `self + other`.
- `def __sub__(self, other):` Allows objects to do `self - other`.
- `def __mul__(self, other):` Allows objects to do `self * other`.

In addition, there is also an interface for sequences:

- `def __len__(self):` Allows objects to do `len(self)`.
- `def __getitem__(self, index):` Allows objects to do `self[i]`.
Questions

1.1 What would Python display?

class A():
    def __init__(self, x):
        self.x = x
    def __repr__(self):
        return self.x
    def __str__(self):
        return self.x * 2

class B():
    def __init__(self):
        print("boo!")
        self.a = []
    def add_a(self, a):
        self.a.append(a)
    def __repr__(self):
        print(len(self.a))
        ret = ""
        for a in self.a:
            ret += str(a)
        return ret

>>> A("one")
one
>>> print(A("one"))
oneone
>>> repr(A("two"))
'two'
>>> b = B()
boo!
>>> b.add_a(A("a"))
>>> b.add_a(A("b"))
>>> b
2
aabb
>>> c = A("c")
>>> b.add_a(c)
>>> str(b)
3

'aabbcc'
1.2 Write the function `is_palindrome` such that it works for any data type that implements the sequence interface.

Assume that the Link class has implemented the `__len__` method and a `__getitem__` method which takes in integers.

```python
def is_palindrome(seq):
    """ Returns True if the sequence is a palindrome. A palindrome is a sequence that reads the same forwards as backwards. 
>>> is_palindrome(Link("l", Link("i", Link("n", Link("k"))))))
False
>>> is_palindrome(["l", "i", "n", "k"])
False
>>> is_palindrome("link")
False
>>> is_palindrome(Link.empty)
True
>>> is_palindrome([])
True
>>> is_palindrome(""")
True
>>> is_palindrome(Link("a", Link("v", Link("a"))))
True
>>> is_palindrome(["a", "v", "a"])
True
>>> is_palindrome("ava")
True
""
    
    for i in range(len(seq)//2):
        if seq[i] != seq[len(seq) - 1 - i]:
            return False
    return True
```
2  Linked Lists

There are many different implementations of sequences in Python. Today, we'll explore the linked list implementation.

A linked list is either an empty linked list, or a Link object containing a first value and the rest of the linked list.

To check if a linked list is an empty linked list, compare it against the class attribute Link.empty:

```python
if link is Link.empty:
    print('This linked list is empty!')
else:
    print('This linked list is not empty!')
```

Implementation

```python
class Link:
    empty = ()

def __init__(self, first, rest=empty):
    assert rest is Link.empty or isinstance(rest, Link)
    self.first = first
    self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_str = ', ' + repr(self.rest)
        else:
            rest_str = ''
        return 'Link({0}{1}).format(repr(self.first), rest_str)

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' ' 
            self = self.rest
        return string + str(self.first) + '>'
```
Questions

2.1 Write a function that takes in a a linked list and returns the sum of all its elements.
You may assume all elements in `lnk` are integers.

```python
def sum_nums(lnk):
    ":=
>>> a = Link(1, Link(6, Link(7)))
>>> sum_nums(a)
14
":=

if lnk == Link.empty:
    return 0
return lnk.first + sum_nums(lnk.rest)
```

2.2 Write a function that takes in a Python list of linked lists and multiplies them
element-wise. It should return a new linked list.

If not all of the `Link` objects are of equal length, return a linked list whose length is
that of the shortest linked list given. You may assume the `Link` objects are shallow
linked lists, and that `lst_of_lnks` contains at least one linked list.

```python
def multiply_lnks(lst_of_lnks):
    ":=
>>> a = Link(2, Link(3, Link(5)))
>>> b = Link(6, Link(4, Link(2)))
>>> c = Link(4, Link(1, Link(0, Link(2))))
>>> p = multiply_lnks([a, b, c])
>>> p.first
48
>>> p.rest.first
12
>>> p.rest.rest.rest is Link.empty
True
":="

Recursive solution:

```
product = 1
for lnk in lst_of_lnks:
    if lnk is Link.empty:
        return Link.empty
    product *= lnk.first
lst_of_lnks_rests = [lnk.rest for lnk in lst_of_lnks]
return Link(product, multiply_lnks(lst_of_lnks_rests))
```

For our base case, if we detect that any of the lists in the list of `Links` is empty, we
can return the empty linked list as we’re not going to multiply anything.
Otherwise, we compute the product of all the firsts in our list of Links. Then, the subproblem we use here is the rest of all the linked lists in our list of Links. Remember that the result of calling `multiply_lnk` will be a linked list! We’ll use the product we’ve built so far as the first item in the returned `Link`, and then the result of the recursive call as the rest of that `Link`.

Iterative solution:

```python
import operator
from functools import reduce
def prod(factors):
    return reduce(operator.mul, factors, 1)

head = Link.empty
tail = head
while Link.empty not in lst_of_lnk:
    all_prod = prod([l.first for l in lst_of_lnk])
    if head is Link.empty:
        head = Link(all_prod)
        tail = head
    else:
        tail.rest = Link(all_prod)
        tail = tail.rest
    lst_of_lnk = [l.rest for l in lst_of_lnk]
return head
```

The iterative solution is a bit more involved than the recursive solution. Instead of building the list “backwards” as in the recursive solution (because of the order that the recursive calls result in, the last item in our list will be finished first), we’ll build the resulting linked list as we go along.

We use `head` and `tail` to track the front and end of the new linked list we’re creating. Our stopping condition for the loop is if any of the `Links` in our list of `Links` runs out of items.

Finally, there’s some special handling for the first item. We need to update both `head` and `tail` in that case. Otherwise, we just append to the end of our list using `tail`, and update `tail`. 
3 Iterators and Generators

An iterable is a data type which contains a collection of values which can be processed one by one sequentially. Some examples of iterables we’ve seen include lists, tuples, strings, and dictionaries. In general, any object that can be iterated over in a for loop can be considered an iterable.

While an iterable contains values that can be iterated over, we need another type of object called an iterator to actually retrieve values contained in an iterable. Calling the iter function on an iterable will create an iterator over that iterable. Each iterator keeps track of its position within the iterable. Calling the next function on an iterator will give the current value in the iterable and move the iterator’s position to the next value.

In this way, the relationship between an iterable and an iterator is analogous to the relationship between a book and a bookmark - an iterable contains the data that is being iterated over, and an iterator keeps track of your position within that data.

Once an iterator has returned all the values in an iterable, subsequent calls to next on that iterable will result in a StopIteration exception. In order to be able to access the values in the iterable a second time, you would have to create a second iterator.

One important application of iterables and iterators is the for loop. We’ve seen how we can use for loops to iterate over iterables like lists and dictionaries.

This only works because the for loop implicitly creates an iterator using the built-in iter function. Python then calls next repeatedly on the iterator, until it raises StopIteration.

The code to the right shows how we can mimic the behavior of for loops using while loops.

Note that most iterators are also iterables - that is, calling iter on them will return an iterator. This means that we can use them inside for loops. However, calling iter on most iterators will not create a new iterator - instead, it will simply return the same iterator.

We can also iterate over iterables in a list comprehension or pass in an iterable to the built-in function list in order to put the items of an iterable into a list.

In addition to the sequences we’ve learned, Python has some built-in ways to create iterables and iterators. Here are a few useful ones:

- range(start, end) returns an iterable containing numbers from start to end-1. If start is not provided, it defaults to 0.
- map(f, iterable) returns a new iterator containing the values resulting from applying f to each value in iterable.
- filter(f, iterable) returns a new iterator containing only the values in iterable for which f(value) returns True.
Questions

3.1 What would Python display? If a StopIteration Exception occurs, write StopIteration, and if another error occurs, write Error.

```python
>>> lst = [6, 1, "a"]
>>> next(lst)
Error

>>> lst_iter = iter(lst)
>>> next(lst_iter)
6

>>> next(lst_iter)
1

>>> next(iter(lst))
6

>>> [x for x in lst_iter]
["a"]
```

Generators

A generator function is a special kind of Python function that uses a yield statement instead of a return statement to report values. When a generator function is called, it returns a generator object, which is a type of iterator. To the right, you can see a function that returns an iterator over the natural numbers.

The yield statement is similar to a return statement. However, while a return statement closes the current frame after the function exits, a yield statement causes the frame to be saved until the next time next is called, which allows the generator to automatically keep track of the iteration state.

Once next is called again, execution resumes where it last stopped and continues until the next yield statement or the end of the function. A generator function can have multiple yield statements.

Including a yield statement in a function automatically tells Python that this function will create a generator. When we call the function, it returns a generator object instead of executing the body. When the generator's next method is called, the body is executed until the next yield statement is executed.

```python
>>> def gen_naturals():
...     current = 0
...     while True:
...         yield current
...         current += 1

>>> gen = gen_naturals()
>>> next(gen)
0
>>> next(gen)
1
```
When `yield from` is called on an iterator, it will `yield` every value from that iterator. It's similar to doing the following:

```python
for x in an_iterator:
    yield x
```

The example to the right demonstrates different ways of computing the same result.

### Questions

3.1 Implement `filter_link`, which takes in a linked list `link` and a function `f` and returns a generator which yields the values of `link` for which `f` returns `True`.

Try to implement this both using a while loop and without using any form of iteration.

```python
def filter_link(link, f):
    
    >>> link = Link(1, Link(2, Link(3)))
    >>> g = filter_link(link, lambda x: x % 2 == 0)
    >>> next(g)
    2
    >>> next(g)
    StopIteration
    >>> list(filter_link(link, lambda x: x % 2 != 0))
    [1, 3]

    while _________________:
        if ________________:
            ________________
            ________________

        ________________

    def filter_link(link, f):
        while link is not Link.empty:
            if f(link.first):
                yield link.first
            link = link.rest

    def filter_no_iter(link, f):
        
        >>> link = Link(1, Link(2, Link(3)))
        >>> list(filter_no_iter(link, lambda x: x % 2 != 0))
        [1, 3]
```

```
if _________________:
    
    return

eelif _________________:
    
    ________________

def filter_no_iter(link, f):
    if link is Link.empty:
        return
eelif f(link.first):
        yield link.first
        yield from filter_no_iter(link.rest, f)

3.2 Implement sum_paths_gen, which takes in a Tree instance t and and returns a generator which yields the sum of all the nodes from a path from the root of a tree to a leaf.

You may yield the sums in any order.

def sum_paths_gen(t):
    '''
    >>> t1 = Tree(5)
    >>> next(sum_paths_gen(t1))
    5
    >>> t2 = Tree(1, [Tree(2, [Tree(3), Tree(4)]), Tree(9)])
    >>> sorted(sum_paths_gen(t2))
    [6, 7, 10]
    '''

    if _________________:
        
        yield ________________

for _________________:
    
    for _________________:
        
        yield ________________

def sum_paths_gen(t):
    if t.is_leaf():
        yield t.label
        for b in t.branches:
for s in sum_paths_gen(b):
    yield s + t.label