1 Mutable Trees

Recall the tree abstract data type: a tree is defined as having a label and some branches. Previously, we implemented the tree abstraction using Python lists. Let’s look at another implementation using objects instead:

```python
class Tree:
    def __init__(self, label, branches=[]):
        for b in branches:
            assert isinstance(b, Tree)
        self.label = label
        self.branches = branches

    def is_leaf(self):
        return not self.branches
```

Notice that with this implementation we can mutate a tree using attribute assignment, which wasn’t possible in the previous implementation using lists.

```python
>>> t = Tree(3, [Tree(4), Tree(5)])
>>> t.label = 5
>>> t.label
5
```

Questions

1.1 What would Python display? If you believe an expression evaluates to a Tree object, write `Tree`.

```python
>>> t0 = Tree(0)
>>> t0.label
```

```python
>>> t0.branches
```

```python
>>> t1 = Tree(0, [1, 2]) # Is this a valid tree?
>>> t2 = Tree(0, [Tree(1), Tree(2, [Tree(3)])])
>>> t2.branches[0]
```

```python
>>> t2.branches[1].branches[0].label
```
1.2 Define a function `make_even` which takes in a tree `t` whose values are integers, and mutates the tree such that all the odd integers are increased by 1 and all the even integers remain the same.

```python
def make_even(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(3)]), Tree(4), Tree(5)])
    >>> make_even(t)
    >>> t.label
    2
    >>> t.branches[0].branches[0].label
    4
    """
```

1.3 Define a function `square_tree(t)` that squares every value in the non-empty tree `t`. You can assume that every value is a number.

```python
def square_tree(t):
    """Mutates a Tree t by squaring all its elements."""
```

1.4 Assuming that every value in `t` is a number, let's define `average(t)`, which returns the average of all the values in `t`.

```python
def average(t):
    """
    Returns the average value of all the nodes in t.
    >>> t0 = Tree(0, [Tree(1), Tree(2, [Tree(3)])])
    >>> average(t0)
    1.5
    >>> t1 = Tree(8, [t0, Tree(4)])
    >>> average(t1)
    3.0
    """
```
2 Mutable Functions in Python

Until now, you’ve been able to access names in parent frames, but you have not been able to modify them. The nonlocal keyword can be used to modify a binding in a parent frame. For example, consider stepper, which uses nonlocal to modify num:

```python
def stepper(num):
    def step():
        nonlocal num  # declares num as a nonlocal name
        num = num + 1  # modifies num in the stepper frame
        return num
    return step

>>> step1 = stepper(10)
>>> step1()  # Modifies and returns num
11
>>> step1()  # num is maintained across separate calls to step
12
>>> step2 = stepper(10)  # Each returned step function keeps its own state
>>> step2()
11
```

As illustrated in this example, nonlocal is useful for maintaining state across different calls to the same function.

However, there are two important caveats with nonlocal names:

- **Global names** cannot be modified using the nonlocal keyword.
- **Names in the current frame** cannot be overridden using the nonlocal keyword. This means we cannot have both a local and nonlocal binding with the same name in a single frame.

Because nonlocal lets you modify bindings in parent frames, we call functions that use it **mutable functions**.
Questions

2.1 Draw the environment diagram for the following code.

```python
def stepper(num):
    def step():
        nonlocal num
        num = num + 1
        return num
    return step

s = stepper(3)
s()
s()
s()
```
Given the definition of `make_shopkeeper` below, draw the environment diagram.

```python
def make_shopkeeper(total_gold):
    def buy(cost):
        nonlocal total_gold
        if total_gold < cost:
            return 'Go farm some more champions'
        total_gold = total_gold - cost
        return total_gold
    return buy

infinity_edge, zeal, gold = 3800, 1100, 3800
shopkeeper = make_shopkeeper(gold - 1000)
shopkeeper(zeal)
shopkeeper(infinity_edge)
```
2.3 Write a function that takes in a number \( n \) and returns a one-argument function. The returned function takes in a function that is used to update \( n \). It prints the updated \( n \) value and returns a function that has the same behavior as itself.

```python
def memory(n):
    """
    >>> f = memory(10)
    >>> f = f(lambda x: x * 2)
    20
    >>> f = f(lambda x: x - 7)
    13
    >>> f = f(lambda x: x > 5)
    True
    """
```
2.4 The bathtub below simulates an epic battle between Finn and Kylo Ren over a populace of rubber duckies. Fill in the body of `ducky` so that all doctests pass.

```python
def bathtub(n):
    ""
    >>> annihilator = bathtub(500) # the force awakens...
    >>> kylo_ren = annihilator(10)
    >>> kylo_ren()
    490 rubber duckies left
    >>> rey = annihilator(-20)
    >>> rey()
    510 rubber duckies left
    >>> kylo_ren()
    500 rubber duckies left
    ""

def ducky_annihilator(rate):
    def ducky():
        return ducky
    return ducky_annihilator
```
3 Mutable Functions in Scheme

We can also create mutable functions in Scheme. In Python, when we want to modify a binding in a parent frame, we declare it to be nonlocal at the start of the function and then assign to the name as normal.

In Scheme, we don’t need to declare which bindings from a parent frame we wish to modify. Instead, we use a new special form call set! when we want to modify an existing binding (regardless of whether that binding exists in the current frame or a parent frame).

Just like the define special form, set! takes in two operands: the symbol to be re-assigned and an expression that should be evaluated and assigned to that symbol.

\[(\text{set!} \ <\text{symbol}> <\text{expression}>)\]

Here’s the same stepper function from earlier, now written in Scheme.

\[
(\text{define} \ (\text{stepper} \ \text{num})
    \begin{align*}
    (\text{define} \ (\text{step}) & \\ \\ (\text{set!} \ \text{num} \ (+ \ \text{num} \ 1)) \\
    & \\ \\ \text{num}) \\
    \text{step}
    \end{align*}
)\]

set! will always modify the most local binding for that symbol that exists. In other words, if the symbol is bound in the current frame, set! works the same as define. Otherwise, it proceeds through parent frames until it finds the symbol, and then re-binds it in that frame. If the binding does not exist anywhere within the current environment, set! will error.

Unlike nonlocal, set! can even modify bindings in the global frame. For example:

\[
\text{scm}> (\text{define} \ \text{count} \ 0)
\]

\[
\text{count}
\]

\[
\text{scm}> (\text{define} \ (\text{increment}) \ (\text{set!} \ \text{count} \ (+ \ \text{count} \ 1)))
\]

\[
\text{increment}
\]

\[
\text{scm}> (\text{increment})
\]

\[
\text{scm}> (\text{increment})
\]

\[
\text{scm}> (\text{increment})
\]

\[
\text{scm}> \text{count}
\]

\[
3
\]

\[
\]
Questions

3.1 Write a procedure `make-piggy-bank`, which returns a one-argument procedure (which we’ll call a piggy bank). A piggy bank starts with nothing inside it. Each time you call it with a positive number, that amount is added to the bank’s total. If you instead pass in number less than or equal to 0, the piggy bank should reset its total to 0 and then return the old total.

```
(define (make-piggy-bank)
```

```
scm> (define piggy-bank (make-piggy-bank))
piggy-bank
scm> (piggy-bank 5) ; add $5
scm> (piggy-bank 10) ; add $10
scm> (piggy-bank 3) ; add $3
scm> (piggy-bank 0) ; dump the money out
18
scm> (piggy-bank 4) ; add $4
scm> (piggy-bank 0) ; dump the money out
4
```