1 Recursion

1.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers \(x\) and \(y\). It returns a list of paths, where each path is a list containing steps to reach \(y\) from \(x\) by repeated incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is \([3, 4, 8, 9]\)

```python
def paths(x, y):
    """Return a list of ways to reach y from x by repeated incrementing or doubling."
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
    if __________________________:
        return ______________________________________________
    elif _________________________:
        return ______________________________________________
    else:
        a = _________________________________________________
        b = _________________________________________________
        return ______________________________________________
```

def paths(x, y):
    if x > y:
        return []
    elif x == y:
        return [[x]]
    else:
a = paths(x + 1, y)
b = paths(x * 2, y)
return [[x] + subpath for subpath in a + b]
1.2 We will now write one of the faster sorting algorithms commonly used, known as *merge sort*. Merge sort works like this:

1. If there is only one (or zero) item(s) in the sequence, it is already sorted!
2. If there are more than one item, then we can split the sequence in half, sort each half recursively, then merge the results, using the *merge* procedure described below. The result will be a sorted sequence.

Using the algorithm described, write a function `mergesort(seq)` that takes an unsorted sequence and sorts it.

Recall the *merge* procedure is as follows:

```python
def merge(s1, s2):
    """Merges two sorted lists """
    if len(s1) == 0:
        return s2
    elif len(s2) == 0:
        return s1
    elif s1[0] < s2[0]:
        return [s1[0]] + merge(s1[1:], s2)
    else:
        return [s2[0]] + merge(s1, s2[1:])

def mergesort(seq):
    if len(seq) <= 1:
        return seq
    else:
        mid = len(seq) // 2
        return merge(mergesort(seq[:mid]),
                     mergesort(seq[mid:]))
```

*Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.*
2 Trees

2.1 Implement `long_paths`, which returns a list of all paths in a tree with length at least \( n \). A path in a tree is a linked list of node values that starts with the root and ends at a leaf. Each subsequent element must be from a child of the previous value’s node. The length of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are listed in order from left to right. See the doctests for some examples.

```python
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least \( n \)."

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    <0 1 2>
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 6 9>
    <0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    ...     print(path)
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14))))])]
    ""

    paths = []
    if n <= 0 and tree.is_leaf():
        paths.append(Link(tree.label))
    for b in tree.branches:
        for path in long_paths(b, n - 1):
            paths.append(Link(tree.label, path))
    return paths
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
Write a function that takes a Tree object and returns the elements at the depth with the most elements.

In this problem, you may find it helpful to use the second optional argument to `sum`, which provides a starting value. All items in the sequence to be summed will be concatenated to the starting value. By default, start will default to 0, which allows you to sum a sequence of numbers. We provide an example of `sum` starting with a list, which allows you to concatenate items in a list.

```python
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...          Tree(4, [Tree(9, [Tree(2)])])])
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
x = [t]

    while ________________________________:
        ____________________________ = sum(_______________________________, [])

        return max(levels, key=______________________________)
```

Main idea: we’ll traverse each level of the tree and keep track of the elements of the levels. After we’re done, we return the level with the most items.

Here, `x` keeps track of the trees in the current level. To get the next level of trees, we take all the branches from all the trees in the current level. The special `sum` call is needed to make sure we get a list of trees, instead of a list of branches (since branches are a list of trees themselves).

Finally, we use `max` with a key to select the list with the longest length from our list of levels.
3  Mutation

3.1 For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```python
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
[1, 2, 23]
>>> dogs[1] = list(dogs)
>>> dogs[1]
[[1, 2, 23], None, [1, 2, 23]]
>>> dogs[0].append(2)
>>> cats
[1, 2, 23, 2]
>>> cats[1::2]
[2, 2]
>>> cats[:3]
[1, 2, 23]
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
Index Error
>>> dogs
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
4 OOP

4.1 (Summer 2015 Final) The TAs are building a social networking website called CS61A+. The TAs plan to represent the network in a class called Network that supports the following method:

- add_friend(user1, user2) adds user1 and user2 to each other’s friends lists. If user1 or user2 are not in the Network, add them to the dictionary of friends.

Help the TAs implement these two methods to make their social networking website popular!

```python
class Network:
    ""
    >>> cs61a_plus = Network()
    >>> cs61a_plus.add_friend('Robert', 'Jeffrey')
    >>> cs61a_plus.friends['Robert']
    ['Jeffrey']
    >>> cs61a_plus.friends['Jeffrey']
    ['Robert']
    >>> cs61a_plus.add_friend('Jessica', 'Robert')
    >>> cs61a_plus.friends['Jessica']
    ['Robert']
    >>> cs61a_plus.add_friend('Jessica', 'Robert')
    >>> cs61a_plus.friends['Jessica']
    ['Robert']
    ""
    def __init__(self):
        self.friends = {}  # Maps users to a list of their friends

    def add_friend(self, user1, user2):
        if user1 not in self.friends:
            self.friends[user1] = []
        if user2 not in self.friends:
            self.friends[user2] = []
        self.friends[user1].append(user2)
        self.friends[user2].append(user1)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
```python
self.friends[user1] = []
if user2 not in self.friends:
    self.friends[user2] = []
self.friends[user1].append(user2)
self.friends[user2].append(user1)
```
CS61A+ turns out to be unpopular. To attract more users, the TAs want to implement a feature that checks if two users have at most \( n \) degrees of separation. Consider the following CS61A+ Network:

```python
self.friends = {
    'Robert': ['Jeffrey', 'Jessica'],
    'Jeffrey': ['Robert', 'Jessica', 'Yulin'],
    'Jessica': ['Robert', 'Jeffrey', 'Yulin'],
    'Yulin': ['Jeffrey', 'Jessica'],
    'Albert': []
}
```

- There is 1 degree of separation between Robert and Jeffrey, because they are direct friends.
- There are 2 degrees of separation between Robert and Yulin (Robert \( \rightarrow \) Jessica \( \rightarrow \) Yulin)
- The degree of separation between Albert and anyone else is undefined, since Albert has no friends.

```python
class Network:
    # Code from previous question

    def degrees(self, user1, user2, n):
        """In these doctests, assume cs61a_plus is a Network with the dictionary of friends described in the example."

        >>> cs61a_plus.degrees('Robert', 'Yulin', 2) # Exactly 2 degrees
        True
        >>> cs61a_plus.degrees('Robert', 'Jessica', 2) # Less than 2 degrees
        True
        >>> cs61a_plus.degrees('Yulin', 'Robert', 1) # More than 1 degree
        False
        >>> cs61a_plus.degrees('Albert', 'Jessica', 10) # No friends!
        False
        """

        if ________________________________:
            return True

        elif ________________________________:
            return False

        for friend in ________________________________:
            if ________________________________:
                return True

        return ________________________________
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
class Network:
    # Code from previous question

def degrees(self, user1, user2, n):
    '''
    >>> cs61a_plus = Network()
    >>> cs61a_plus.friends = {
    ...     'Robert': ['Jeffrey', 'Jessica'],
    ...     'Jeffrey': ['Robert', 'Jessica', 'Yulin'],
    ...     'Jessica': ['Robert', 'Jeffrey', 'Yulin'],
    ...     'Yulin': ['Jeffrey', 'Jessica'],
    ...     'Albert': []
    ... }
    >>> cs61a_plus.degrees('Robert', 'Yulin', 2)  # Exactly 2 degrees
    True
    >>> cs61a_plus.degrees('Robert', 'Jessica', 2)  # Less than 2 degrees
    True
    >>> cs61a_plus.degrees('Yulin', 'Robert', 1)  # More than 1 degree
    False
    >>> cs61a_plus.degrees('Albert', 'Jessica', 10)  # No friends!
    False
    '''
    if user1 == user2:
        return True
    elif n <= 0:
        return False
    for friend in self.friends[user1]:
        if self.degrees(friend, user2, n - 1):
            return True
    return False

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
5.1 Write a recursive function `flip_two` that takes as input a linked list `lnk` and mutates `lnk` so that every pair is flipped.

```python
def flip_two(lnk):
    """
    >>> one_lnk = Link(1)
    >>> flip_two(one_lnk)
    >>> one_lnk
    Link(1)
    >>> lnk = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> flip_two(lnk)
    >>> lnk
    Link(2, Link(1, Link(4, Link(3, Link(5)))))
    """

    Recursive solution:
    
    ```python
    if lnk is Link.empty or lnk.rest is Link.empty:
        return
    lnk.first, lnk.rest.first = lnk.rest.first, lnk.first
    flip_two(lnk.rest.rest)
    ```
    
    If there's only a single item (or no item) to flip, then we're done.

    Otherwise, we swap the contents of the first and second items in the list. Since we've handled the first two items, we then need to recurse on

    Although the question explicitly asks for a recursive solution, there is also a fairly similar iterative solution:

    ```python
    while lnk is not Link.empty and lnk.rest is not Link.empty:
        lnk.first, lnk.rest.first = lnk.rest.first, lnk.first
        lnk = lnk.rest.rest
    ```

    We will advance `lnk` until we see there are no more items or there is only one more `Link` object to process. Processing each `Link` involves swapping the contents of the first and second items in the list (same as the recursive solution).

    Notice that the code is remarkably similar to the recursive implementation of `flip_two`.

    Video walkthrough

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*Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.*
6 Generators

6.1 Write a generator function that yields functions that are repeated applications of a one-argument function f. The first function yielded should apply f 0 times (the identity function), the second function yielded should apply f once, etc.

```python
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> func = repeated(double)
    >>> identity = next(func)
    >>> double = next(func)
    >>> quad = next(func)
    >>> oct = next(func)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...     zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    """
    g = ________________________________

    while True:
        ________________________________
        ________________________________
        ________________________________
        ________________________________

    def repeated(f):
        g = lambda x: x
        while True:
            yield g
            g = (lambda g: lambda x: f(g(x)))(g)

Video walkthrough

6.2 Ben Bitdiddle proposes the following alternate solution. Does it work?

```python
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = (lambda g: lambda x: f(g(x)))(g)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
This solution does not work. The value g changes with each iteration so the bodies of the lambdas yielded change as well.

6.3 Implement **accumulate**, which takes in an iterable and a function f and yields each accumulated value from applying f to the running total and the next element.

```python
from operator import add, mul

def accumulate(iterable, f):
    
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    
    it = iter(iterable)

    total = next(it)
    yield total
    for element in it:
        total = f(total, element)
        yield total
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
7 Scheme

7.1 Write a function that takes a procedure and applies to every element in a given nested list.

The result should be a nested list with the same structure as the input list, but with each element replaced by the result of applying the procedure to that element.

Use the built-in list? procedure to detect whether a value is a list.

(define (deep-map fn lst)
  (cond ((null? lst) lst)
    ((list? (car lst)) (cons (deep-map fn (car lst)) (deep-map fn (cdr lst))))
    (else (cons (fn (car lst)) (deep-map fn (cdr lst))))))

scm> (deep-map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-map (lambda (x) (* x x)) '((1 (4) 5) 9))
(1 ((16) 25) 81)

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.