1 Mutation

1.1 For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```python
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
[1, 2, 23]
>>> dogs[1] = list(dogs)
>>> dogs[1]
[[1, 2, 23], None, [1, 2, 23]]
>>> dogs[0].append(2)
>>> cats
[1, 2, 23, 2]
>>> cats[1::2]
[2, 2]
>>> cats[:3]
[1, 2, 23]
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
Index Error
>>> dogs
```
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
2 Recursion

2.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers `x` and `y`. It returns a list of paths, where each path is a list containing steps to reach `y` from `x` by repeated incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is `[3, 4, 8, 9]

```python
def paths(x, y):
    """Return a list of ways to reach y from x by repeated incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    ""
    if ________________________:
        return ________________________________
    elif _________________________:
        return ________________________________
    else:
        a = _________________________________
        b = _________________________________
        return _______________________________
```

```python
def paths(x, y):
    if x > y:
        return []
    elif x == y:
        return [[x]]
    else:
        a = paths(x + 1, y)
        b = paths(x * 2, y)
        return [[x] + subpath for subpath in a + b]
```
3 Trees

3.1 Implement long_paths, which returns a list of all paths in a tree with length at least \( n \). A path in a tree is a linked list of node values that starts with the root and ends at a leaf. Each subsequent element must be from a child of the previous value’s node. The length of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are listed in order from left to right. See the doctests for some examples.

```python
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least n."

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])]), Tree(9)])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...     print(path)
    ...<0 1 2>
    ...<0 1 3 4>
    ...<0 1 3 4>
    ...<0 1 3 5>
    ...<0 6 7 8>
    ...<0 6 9>
    ...<0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...     print(path)
    ...<0 1 3 4>
    ...<0 1 3 4>
    ...<0 1 3 5>
    ...<0 6 7 8>
    ...<0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14)))))]
    """

    paths = []
    if n <= 0 and tree.is_leaf():
        paths.append(Link(tree.label))
    for b in tree.branches:
        for path in long_paths(b, n - 1):
            paths.append(Link(tree.label, path))
    return paths
```

4 \textbf{Streams}

4.1 Write a function \texttt{merge} that takes 2 sorted streams \texttt{s1} and \texttt{s2}, and returns a new sorted stream which contains all the elements from \texttt{s1} and \texttt{s2}. Assume that both \texttt{s1} and \texttt{s2} have infinite length.

\begin{verbatim}
(define (merge s1 s2)
  (if ________________________________
      ________________________________
      ________________________________
      ________________________________
      ________________________________)

(define (merge s1 s2)
  (if (< (car s1) (car s2))
      (cons-stream (car s1) (merge (cdr-stream s1) s2))
      (cons-stream (car s2) (merge s1 (cdr-stream s2))))))

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4.2 (Adapted from Fall 2014) Implement \texttt{cycle} which returns a stream repeating the digits 1, 3, 0, 2, and 4, forever. Write \texttt{cons-stream} only once in your solution!

\textbf{Hint}: \((3+2) \mod 5 = 0\).

\begin{verbatim}
(define (cycle start)
  ________________________________)

(define (cycle start)
  (cons-stream start (cycle (modulo (+ start 2) 5))))

Video walkthrough
\end{verbatim}
5 Generators

5.1 Implement `accumulate`, which takes in an iterable and a function `f` and yields each accumulated value from applying `f` to the running total and the next element.

```python
from operator import add, mul

def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    it = iter(iterable)
    total = next(it)
    yield total
    for element in it:
        total = f(total, element)
        yield total
```
5.2 Write a generator function that yields functions that are repeated applications of a one-argument function \( f \). The first function yielded should apply \( f \) 0 times (the identity function), the second function yielded should apply \( f \) once, etc.

```python
def repeated(f):
    
    >>> double = lambda x: 2 * x
    >>> func = repeated(double)
    >>> identity = next(func)
    >>> double = next(func)
    >>> quad = next(func)
    >>> oct = next(func)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...     zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    
```

5.3 Ben Bitdiddle proposes the following alternate solution. Does it work?

```python
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = (lambda g: lambda x: f(g(x)))(g)

Video walkthrough

```
This solution does not work. The value of changes with each iteration so the bodies of the lambdas yielded change as well.
SQL

6.1 You’re starting a new job at an animal shelter, and you’ve been tasked with keeping track of all the cats that are up for adoption!

We’ll start with an empty table:

```
CREATE TABLE cats(name, weight DEFAULT 1, notes DEFAULT "meow");
```

(a) What would SQL display?

```
sqlite> INSERT INTO cats(name) VALUES ("Tom"), ("Whiskers");
sqlite> SELECT * FROM cats;

<table>
<thead>
<tr>
<th>Name</th>
<th>Weight</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom</td>
<td>1</td>
<td>meow</td>
</tr>
<tr>
<td>Whiskers</td>
<td>1</td>
<td>meow</td>
</tr>
</tbody>
</table>
```

```
sqlite> INSERT INTO cats VALUES
     ...> ("Mittens", 2, "Actually likes shoes"),
     ...> ("Rascal", 4, "Prefers to associate with dogs"),
     ...> ("Magic", 2, "Expert at card games");
sqlite> SELECT * FROM cats ORDER BY weight, name;

<table>
<thead>
<tr>
<th>Name</th>
<th>Weight</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom</td>
<td>1</td>
<td>meow</td>
</tr>
<tr>
<td>Whiskers</td>
<td>1</td>
<td>meow</td>
</tr>
<tr>
<td>Magic</td>
<td>2</td>
<td>Expert at card games</td>
</tr>
<tr>
<td>Mittens</td>
<td>2</td>
<td>Actually likes shoes</td>
</tr>
<tr>
<td>Rascal</td>
<td>4</td>
<td>Prefers to associate with dogs</td>
</tr>
</tbody>
</table>
```

```
sqlite> UPDATE cats SET notes = "A cat" WHERE notes = "meow";
sqlite> SELECT name FROM cats WHERE notes = "A cat";

<table>
<thead>
<tr>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom</td>
</tr>
<tr>
<td>Whiskers</td>
</tr>
</tbody>
</table>
```

(b) Cats of different weights require different quantities of food. We have the following table:

```
CREATE TABLE food AS
  SELECT 1 AS cat_weight, 0.5 AS amount UNION
  SELECT 2 , 2.5 UNION
  SELECT 3 , 4.0 UNION
  SELECT 4 , 4.5;
```

Write a query that calculates the total amount of food required to feed all the cats (this should work for any table of cats, not just the one we created above). In our example, we have two cats of weight 1, two cats of weight 2, and one cat of weight 4. The total food required is \(2 \times 0.5 + 2 \times 2.5 + 1 \times 4.5 = 10.5\).
Specifying the table name in the **WHERE** clause here is not necessary and was added just for clarity.

```sql
SELECT SUM(amount) FROM cats, food WHERE cats.weight = food.cat_weight;
```

## 7 Macros

### 7.1 Using macros, let's make a new special form, **when**, that has the following structure:

```
(when <condition>
    (<expr1> <expr2> <expr3> ...))
```

If the condition is not false (a truthy expression), all the subsequent operands are evaluated in order and the value of the last expression is returned. Otherwise, the entire **when** expression evaluates to **okay**.

```
(scm> (when (= 1 0) ((/ 1 0) 'error))
    okay
    scm> (when (= 1 1) ((print 6) (print 1) 'a))
    6
    1
    a
```

(a) Fill in the skeleton below to implement this without using quasiquotes.

```
(define-macro (when condition exprs)
    (list 'if ______________))
```

```
(define-macro (when condition exprs)
    (list 'if ______________
         ,condition,
         ,begin,
         ,exprs)
    'okay)
```

(b) Now, implement the macro using quasiquotes.

```
(define-macro (when condition exprs)
    '(if ______________))
```

```
(define-macro (when condition exprs)
    '(if ,condition,
       ,(cons 'begin exprs)
       'okay))
```

### 7.2 Write a macro called **zero-cond** that takes in a list of clauses, where each clause is a two-element list containing two expressions, a predicate and a corresponding
result expression. All predicates evaluate to a number. The macro should return the value of the expression corresponding to the first true predicate, treating 0 as a false value.

```
scm> (zero-cond
    ((0 'result1)
     ((- 1 1) 'result2)
     ((* 1 1) 'result3)
     (2 'result4)))
```

result3

```
(define-macro (zero-cond clauses)
  (cons 'cond
    (map (lambda (clause)
          (cons '(not (= 0 ,(car clause))) (cdr clause)))
         clauses))
```

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