1 Functions
Questions

1.1 Determine what the Python interpreter will output given the following lines of code.

```python
>>> from operator import add, mul
>>> mul(add(5, 6), 8)
88
>>> print('x')
x
>>> y = print('x')
x
>>> print(y)
None
>>> print(add(4, 2), print('a'))
a
6 None
```

1.2 Determine what the Python interpreter will output given the following lines of code.

```python
>>> def foo(x):
    print(x)
    return x + 1

>>> def bar(y, x):
    print(x - y)

>>> foo(3)
3
4
>>> bar(3)
```
Functions, Control, Environments, HOFs

Error

>>> bar(6, 1)

-5

>>> bar(foo(10), 11)

10

0

2 Control

Questions

2.1 Which numbers will be printed after executing the following code?

```python
n = 0
if n:
    print(1)
elif n < 2:
    print(2)
else:
    print(3)
print(4)
```

2

4

2.2 WWPD (What would Python Display) after evaluating each of the following expressions?

```python
>>> 0 and 1 / 0
```

0

```python
>>> 6 or 1 or "a" or 1 / 0
```

6

```python
>>> 6 and 1 and "a" and 1 / 0
```

Error

```python
>>> print(print(4) and 2)
```

4

None

```python
>>> not True and print("a")
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
2.3 Define a function, `count_digits`, which takes in an integer, `n`, and counts the number of digits in that number.

```python
def count_digits(n):
    ...
    >>> count_digits(4)
    1
    >>> count_digits(12345678)
    8
    >>> count_digits(0)
    0
    ...

    count = 0
    while n > 0:
        count += 1
        n = n // 10
    return count
```

2.4 Define a function, `count_matches`, which takes in two integers `n` and `m`, and counts the number of digits that match.

```python
def count_matches(n, m):
    ...
    >>> count_matches(10, 30)
    1
    >>> count_matches(12345, 23456)
    0
    >>> count_matches(121212, 123123)
    2
    >>> count_matches(111, 11) # only one's place matches
    2
    >>> count_matches(101, 10) # no place matches
    0
    ...

    matches = 0
    while n > 0 and m > 0:
        if n % 10 == m % 10:
            matches += 1
        n, m = n // 10, m // 10
    return matches
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
3 Environment Diagrams

Questions

3.1 Draw the environment diagram for evaluating the following code

```python
def f(x):
    return y + x

y = 10
f(8)
```

Solution: https://goo.gl/rZnzaM

3.2 Draw the environment diagram for evaluating the following code

```python
def dessef(a, b):
    c = a + b
    b = b + 1

b = 6
dessef(b, 4)
```

Solution: https://goo.gl/4m3NRD
3.3 Draw the environment diagram for evaluating the following code

```python
def foo(x, y):
    foo = bar
    return foo(bar(x, x), y)

def bar(z, x):
    return z + y

y = 5
foo(1, 2)
```

Solution: https://goo.gl/7Kcx6n

3.4 Draw the environment diagram for evaluating the following code

```python
def spain(japan, iran):
    def world(cup, egypt):
        return japan-poland
    return iran(world(iran, poland))

def saudi(arabia):
    return japan + 3

japan, poland = 3, 7
spain(poland+1, saudi)
```

Solution: https://goo.gl/iddW49
3.5 Draw the environment diagram for evaluating the following code

cap = 9
hulk = 3

```python
def marvel(cap, thor, avengers):
    marvel = avengers
    iron = hulk + cap
    if thor > cap:
        def marvel(cap, thor, avengers):
            return iron
    else:
        iron = hulk
    return marvel(thor, cap, marvel)

def iron(man):
    hulk = cap - 1
    return hulk

marvel(cap, iron(3), marvel)
```

Solution: https://goo.gl/sofcq2
4 Higher Order Functions

Questions

4.1 What do lambda expressions do? Can we write all functions as lambda expressions? In what cases are lambda expressions useful?

Lambda expressions create functions. When a lambda expression is evaluated, it produces a function. We often use lambdas to create short anonymous functions that we won’t need for too long.

We can’t write all functions as lambda expressions because lambda functions all have to have ?return? statements and they can’t contain very complex multi-line expressions.

4.2 Determine if each of the following will error:

```python
>>> 1/0
Error
>>> boom = lambda: 1/0
No error, since we don’t evaluate the body of the lambda when we define it.
>>> boom()
Error
```

4.3 Express the following lambda expression using a `def` statement, and the `def` statement using a lambda expression.

```python
pow = lambda x, y: x**y
def pow(x, y):
    return x**y
def foo(x):
    def f(y):
        def g(z):
            return x + y * z
        return g
    return f
foo = lambda x: lambda y: lambda z: x + y * z
```
4.4 Draw Environment Diagrams for the following lines of code

```python
square = lambda x: x * x
higher = lambda f: lambda y: f(f(y))
higher(square)(5)
```

**Solution:** [https://goo.gl/LATqV9](https://goo.gl/LATqV9)

```python
a = (lambda f, a: f(a))(lambda b: b * b, 2)
```

**Solution:** [https://goo.gl/TyriuP](https://goo.gl/TyriuP)
4.5 Write `make_skipper`, which takes in a number `n` and outputs a function. When this function takes in a number `x`, it prints out all the numbers between 0 and `x`, skipping every `n`th number (meaning skip any value that is a multiple of `n`).

```python
def make_skipper(n):
    
    >>> a = make_skipper(2)
    >>> a(5)
    1
    3
    5
    
    def skipper(x):
        for i in range(x + 1):
            if i % n != 0:
                print(i)
        return skipper
```
5 Extra Questions

Questions

5.1 Define a function, ordered_digits, which takes in a positive integer, x, and returns True if the (base 10) digits of x are in non-decreasing order, and False otherwise.

```python
def ordered_digits(x):
    
    >>> ordered_digits(5)
    True
    >>> ordered_digits(11)
    True
    >>> ordered_digits(127)
    True
    >>> ordered_digits(1357)
    True
    >>> ordered_digits(21)
    False
    >>> result = ordered_digits(1375) # Return, don't print
    >>> result
    False

    last = x % 10
    val = x // 10
    while x > 0 and last >= x % 10:
        last = x % 10
        x = x // 10
    return x == 0
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
5.2 Define a function, cycle, which takes in three functions, f1, f2, f3, and returns a function that takes in an integer n and returns a function that takes in an integer x, and returns the result of f1(x) the first time it’s called, f2(x) the second time it’s called, f3(x) the third time it’s called, and then cycles back to f1(x) the fourth time it’s called, and so on.

```python
def cycle(f1, f2, f3):
    """Returns a function that is itself a higher-order function."
    def ret(x):
        i = 0
        while i < n:
            if i % 3 == 0:
                x = f1(x)
            elif i % 3 == 1:
                x = f2(x)
            else:
                x = f3(x)
            i += 1
        return x
    return ret
return ret_fn
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
5.3 Define a function, is_palindrome, which takes in an integer, \( n \), and returns True if \( n \) is a palindrome and False otherwise.

```python
def is_palindrome(n):
    """
    Fill in the blanks '_____' to check if a number is a palindrome.
    >>> is_palindrome(12321)
    True
    >>> is_palindrome(42)
    False
    >>> is_palindrome(2015)
    False
    >>> is_palindrome(55)
    True
    """
    x, y = n, 0
    f = lambda: _________________________________
    while x > 0:
        x, y = _____________, f()
    return y == n
x, y = n, 0
f = lambda: y * 10 + x % 10
while x > 0:
    x, y = x // 10, f()
return y == n
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.