1 Tail Recursion

1.1 For the following procedures, determine whether or not they are tail recursive. If they are not, write why not and rewrite the function to be tail recursive on the right.

; Multiplies x by y
(define (mult x y)
  (if (= 0 y)
      0
      (+ x (mult x (- y 1))))

; Always evaluates to true
; assume n is positive
(define (true1 n)
  (if (= n 0)
      #t
      (and #t (true1 (- n 1))))

; Always evaluates to true
; assume n is positive
(define (true2 n)
  (if (= n 0)
      #t
      (or (true2 (- n 1)) #f))

; Returns true if x is in lst
(define (contains lst x)
  (cond
    ((null? lst) #f)
    ((equal? (car lst) x) #t)
    ((contains (cdr lst) x) #t)
    (else #f)))
1.2 Tail recursively implement \texttt{sum-satisfied-k} which, given an input list \texttt{lst}, a predicate procedure \texttt{f} which takes in one argument, and an integer \texttt{k}, will return the sum of the first \texttt{k} elements that satisfy \texttt{f}. If there are not \texttt{k} such elements, return \texttt{0}.

; Doctests
\begin{verbatim}
scm> (define lst '(1 2 3 4 5 6))
scm> (sum-satisfied-k lst even? 2) ; 2 + 4
 6
scm> (sum-satisfied-k lst (lambda (x) (= 0 (modulo x 3))) 10)
 0
scm> (sum-satisfied-k lst (lambda (x) #t) 0)
 0
\end{verbatim}

\begin{verbatim}
(define (sum-satisfied-k lst f k)
  \ldots)
\end{verbatim}

1.3 Tail-recursively implement \texttt{remove-range} which, given one input list \texttt{lst}, and two nonnegative integers \texttt{i} and \texttt{j}, returns a new list containing the elements of \texttt{lst} except the ones from index \texttt{i} to index \texttt{j}. You may assume \texttt{j} > \texttt{i}, and \texttt{j} is less than the length of the list. (Hint: you may want to use the built-in \texttt{append} function)

; Doctests
\begin{verbatim}
scm> (append '(1 2) '(3 4) '(5 6))
(1 2 3 4 5 6)
scm> (remove-range '(0 1 2 3 4) 1 3)
(0 4)
\end{verbatim}

\begin{verbatim}
(define (remove-range lst i j)
  \ldots)
\end{verbatim}

\textit{Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.}
Check your understanding

- Why aren’t all subexpression evaluations tail-recursive? For instance, why isn’t the evaluation of (+ 4 5) as part of evaluating (+ 1 (+ 2 3) (+ 4 5)) tail recursive, even though it’s the last expression in the summation?

- Given a function (f lst) that acts over a list that has a single recursive call of the form (f (cdr lst)), what would be a general approach for rewriting it tail-recursively?
2 Interpreters

2.1 Determine the number of calls to scheme_eval and the number of calls to scheme_apply for the following expressions. Use the visualizer at code.cs61a.org if you’re not sure how an expression is evaluated.

```scheme
> (+ 1 2)
3

> (if 1 (+ 2 3) (/ 1 0))
5

> (or #f (and (+ 2 3) 'apple) (- 5 2))
apple

> (define (add x y) (+ x y))
add
> (add (- 5 3) (or 0 2))
2
```

Check your understanding

- When a Scheme interpreter evaluates a combination of the form \((a \ b \ c \ d \ e)\), when does it evaluate \(a\)? Does it do so when \(a\) evaluates to a user-defined function? What about a builtin procedure? What if it is a keyword for a special form?

- What happens when we redefine a builtin procedure, like #*[+]? For instance, if we run (define + -), and then (+ 1 2), what do we get? What about if we overwrite a keyword corresponding to a special form?
3 Macros

3.1 What will Scheme display? If you think it errors, write Error

> (define-macro (doerror) (/ 1 0))

> (doerror)

> (define x 5)

> (define-macro (evaller y) (list (list 'lambda '(x) 'x) y))

> (evaller 2)

3.2 Consider a new special form, **when**, that has the following structure:

\[
\text{(when} \ <\text{condition}> \ <\text{expr1}> \ <\text{expr2}> \ <\text{expr3}> \ \ldots \ \text{)}
\]

If the condition is not false (a truthy expression), all the subsequent operands are evaluated in order and the value of the last expression is returned. Otherwise, the entire **when** expression evaluates to **okay**.

scm> (when (= 1 0) (/ 1 0) 'error)
okay
scm> (when (= 1 1) (print 6) (print 1) 'a)
6
1
a

Create this new special form using a macro. Recall that putting a dot before the last formal parameter allows you to pass any number of arguments to a procedure, a list of which will be bound to the parameter, similar to (*args) in Python.

; implement when without using quasiquotes
(define-macro (when condition . exprs)
  (list 'if _________________________________))

; implement when using quasiquotes
(define-macro (when condition . exprs)
  '(if _________________________________))

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