Pick someone in your group to **join Discord**. It’s fine if multiple people join, but one is enough.

Now switch to Pensieve:

- **Everyone**: Go to [discuss.pensieve.co](http://discuss.pensieve.co) and log in with your @berkeley.edu email, then enter your group number. (Your group number is the number of your Discord channel.)

Once you’re on Pensieve, you don’t need to return to this page; Pensieve has all the same content (but more features). If for some reason Pensieve doesn’t work, return to this page and continue with the discussion.

Post in the [#help](http://Discord) channel on Discord if you have trouble.

## Getting Started

Say your name and share a food that you really liked as a child. (It’s ok if you still like that food now.)

**Suggestion**: After Midterm 1, some students are looking for more effective ways to study. One great option is to meet up with your discussion group outside of class to review practice problems together. Now is a great time to schedule a time and place for some extra group practice of old Midterm 1 questions. This is optional and not everyone needs to come, but if there are Midterm 1 topics that haven’t totally clicked yet, this weekend is a perfect time to review them.

Everything in this course builds on prior topics, and it’s going to be hard to keep up if you don’t have a solid understanding of Midterm 1 material.

Remember, it’s ok if someone hasn’t learned everything yet and needs more time to master the course material. The whole point of the course is for students to learn things they don’t already know. Please support each other in the process.

## Recursion

Ok, time to discuss problems! Remember to work together. Everyone in the group should understand a solution before the group moves on. Many students find this discussion challenging. Everything gets easier with practice.

**VERY IMPORTANT**: In this discussion, don’t check your answers until your whole group is sure that the answer is right. Figure things out and check your work by thinking about what your code will do. Your goal should be to have all checks pass the first time you run them! If you need help, ask.

**Q1: Swipe**

Implement **swipe**, which prints the digits of argument `n`, one per line, first backward then forward. The left-most digit is printed only once. Do not use **while** or **for** or **str**. (Use recursion, of course!)
def swipe(n):
    """Print the digits of n, one per line, first backward then forward."

    >>> swipe(2837)
    7
    3
    8
    2
    8
    3
    7
    """
    if n < 10:
        print(n)
    else:
        print(n % 10)
        swipe(n // 10)
        print(n % 10)

First print the first line of the output, then make a recursive call, then print the last line of the output.
Q2: Skip Factorial

Define the base case for the `skip_factorial` function, which returns the product of every other positive integer, starting with \( n \).

```python
def skip_factorial(n):
    """Return the product of positive integers \( n * (n - 2) * (n - 4) \ldots \)
    ""

    >>> skip_factorial(5)  # 5 * 3 * 1
    15
    >>> skip_factorial(8)  # 8 * 6 * 4 * 2
    384
    """
    if n <= 2:
        return n
    else:
        return n * skip_factorial(n - 2)
```

If \( n \) is even, then the base case will be 2. If \( n \) is odd, then the base case will be 1. Try to write a condition that handles both possibilities.

*Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.*
Q3: Is Prime

Implement `is_prime` that takes an integer `n` greater than 1. It returns `True` if `n` is a prime number and `False` otherwise. Try following the approach below, but implement it recursively without using a `while` (or `for`) statement.

```python
def is_prime(n):
    assert n > 1
    i = 2
    while i < n:
        if n % i == 0:
            return False
        i = i + 1
    return True
```

You will need to define another “helper” function (a function that exists just to help implement this one). Does it matter whether you define it within `is_prime` or as a separate function in the global frame? Try to define it to take as few arguments as possible.

```python
def is_prime(n):
    """Returns True if n is a prime number and False otherwise."
    >>> is_prime(2)
    True
    >>> is_prime(16)
    False
    >>> is_prime(521)
    True
    """
    def check_all(i):
        """Check whether no number from i up to n evenly divides n.""
        if i == n:  # could be replaced with i > (n ** 0.5)
            return True
        elif n % i == 0:
            return False
        return check_all(i + 1)
    return check_all(2)
```

Define an inner function that checks whether some integer between `i` and `n` evenly divides `n`. Then you can call it starting with `i=2`:

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
def is_prime(n):
    def f(i):
        if n % i == 0:
            return _____
        elif _____:
            return _____
        else:
            return f(____)
    return f(2)

Come up with a one sentence docstring for the helper function that describes what it does. Don’t just write, “it helps implement is_prime.” Instead, describe its behavior. When you’re done, paste the text of that docstring in your group’s channel’s text chat.
Q4: Recursive Hailstone

Recall the hailstone function from Homework 1. First, pick a positive integer $n$ as the start. If $n$ is even, divide it by 2. If $n$ is odd, multiply it by 3 and add 1. Repeat this process until $n$ is 1. Complete this recursive version of hailstone that prints out the values of the sequence and returns the number of steps.

```python
def hailstone(n):
    ""
    Print out the hailstone sequence starting at n,
    and return the number of elements in the sequence.
    >>> a = hailstone(10)
    10
    5
    16
    8
    4
    2
    1
    >>> a
    7
    >>> b = hailstone(1)
    1
    >>> b
    1
    ""
    print(n)
    if n % 2 == 0:
        return even(n)
    else:
        return odd(n)
def even(n):
    return 1 + hailstone(n // 2)
def odd(n):
    if n == 1:
        return 1
    else:
        return 1 + hailstone(3 * n + 1)
```

An even number is never a base case, so even always makes a recursive call to hailstone and returns one more than the length of the rest of the hailstone sequence.

An odd number might be 1 (the base case) or greater than one (the recursive case). Only the recursive case should call hailstone.

Once your group has converged on a solution, it’s time to practice your ability to describe your own code. Pick a presenter, then send a message to the discuss-queue channel with the @discuss tag, your discussion group number, and the message “Hailing all course staff!” and a member of the course staff will join your voice channel to hear your
description.

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
Recursion

Document the Occasion

Please all fill out the attendance form (one submission per person per week).

Extra Challenge

You’ll need your whole discussion group for this question. At least try it out. You might have fun. We’ll review the question in lecture on Friday.

Q5: Sevens

The Game of Sevens: Players in a circle count up from 1 in the clockwise direction. (The starting player says 1, the player to their left says 2, etc.) If a number is divisible by 7 or contains a 7 (or both), switch directions. Numbers must be said on the beat at 60 beats per minute. If someone says a number when it’s not their turn or someone misses the beat on their turn, the game ends.

For example, 5 people would count to 20 like this:

<table>
<thead>
<tr>
<th>Player 1 says 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2 says 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 3 says 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 4 says 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 5 says 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1 says 6</td>
<td></td>
<td># All the way around the circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2 says 7</td>
<td></td>
<td># Switch to counterclockwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1 says 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 5 says 9</td>
<td></td>
<td># Back around the circle counterclockwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 4 says 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 3 says 11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2 says 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1 says 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 5 says 14</td>
<td></td>
<td># Switch back to clockwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1 says 15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2 says 16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 3 says 17</td>
<td></td>
<td># Switch back to counterclockwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2 says 18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1 says 19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 5 says 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Play a few games. Post the highest score your group reached on Discord.

Then, implement sevens which takes a positive integer $n$ and a number of players $k$. It returns which of the $k$ players says $n$. You may call has_seven.

An effective approach to this problem is to simulate the game, stopping on turn $n$. The implementation must keep track of the final number $n$, the current number $i$, the player who will say $i$, and the current direction that determines the next player (either increasing or decreasing). It works well to use integers to represent all of these, with direction switching between 1 (increase) and -1 (decreasing).

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
def sevens(n, k):
    """Return the (clockwise) position of who says n among k players."

    >>> sevens(2, 5)
    2
    >>> sevens(6, 5)
    1
    >>> sevens(7, 5)
    2
    >>> sevens(8, 5)
    1
    >>> sevens(9, 5)
    5
    >>> sevens(18, 5)
    2
    """
    def f(i, who, direction):
        if i == n:
            return who
        if i % 7 == 0 or has_seven(i):
            direction = -direction
            who = who + direction
        if who > k:
            who = 1
        if who < 1:
            who = k
        return f(i + 1, who, direction)
    return f(1, 1, 1)

def has_seven(n):
    if n == 0:
        return False
    elif n % 10 == 7:
        return True
    else:
        return has_seven(n // 10)

First check if \(i\) is a multiple of 7 or contains a 7, and if so, switch directions. Then, add the direction to \(who\) and ensure that \(who\) has not become smaller than 1 or greater than \(k\).

Q6: Karel the Robot

Karel the robot starts in the corner of an \(n\) by \(n\) square for some unknown number \(n\). Karel responds to only four functions: - `move()` moves Karel one square forward if there is no wall in front of Karel and errors if there is. - `turn_left()` turns Karel 90 degrees to the left. - `front_is_blocked()` returns whether there is a wall in front of Karel. - `front_is_clear()` returns whether there is no wall in front of Karel.

*Note: This worksheet is a problem bank—most TA's will not cover all the problems in discussion section.*
Implement a `main()` function that will leave Karel stopped halfway in the middle of the bottom row. For example, if the square is 7 x 7 and Karel starts in position (1, 1), the bottom left, then Karel should end in position (1, 4) (three steps from either side on the bottom row). Karel can be facing in any direction at the end. If the bottom row length is even, Karel can stop in either position (1, \( \frac{n}{2} \)) or (1, \( \frac{n}{2} + 1 \)).

**Important** You can only write `if` or `if/else` statements and function calls in the body of `main()`. You may not write assignment statements, `def` statements, lambda expressions, or `while/for` statements.

For every two steps forward, take one step back to end up in the middle.