CS 61A Structure and Interpretation of Computer Programs
Fall 2020

Midterm 2

INSTRUCTIONS

This is your exam. Complete it either at exam.cs61a.org or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address `<EMAILADDRESS>`. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with **circular bubbles**, you should select exactly *one* choice.

- ○ You must choose either this option
- ○ Or this one, but not both!

For questions with **square checkboxes**, you may select *multiple* choices.

- ☐ You could select this choice.
- ☐ You could select this one too!

You may start your exam now. Your exam is due at <DEADLINE> Pacific Time. Go to the next page to begin.

Preliminaries

You can complete and submit these questions before the exam starts.

(a) What is your full name?

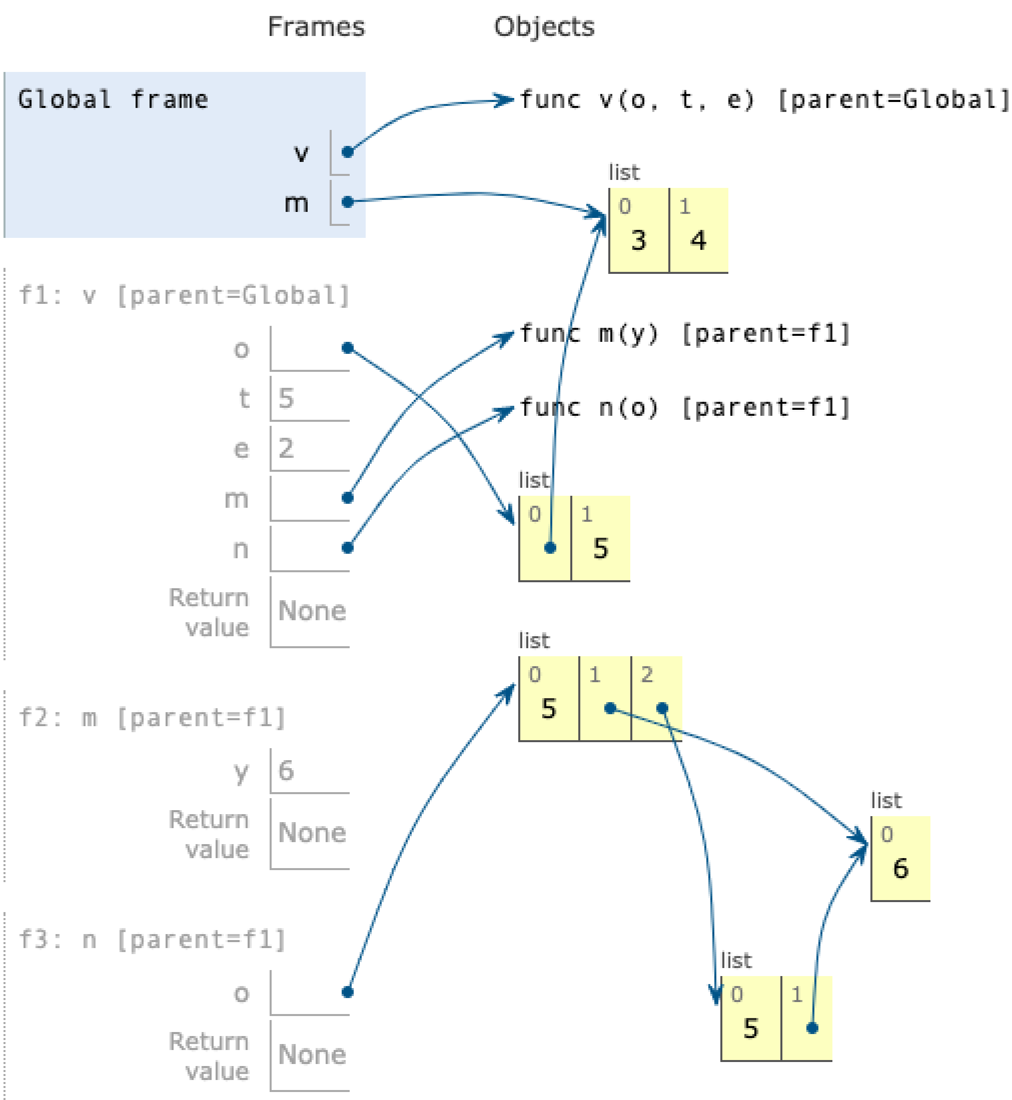
(b) What is your student ID number?

1. (8 points) Political Environment

Fill in each blank in the code example below so that executing it would generate the following environment diagram.

RESTRICTIONS. You must use all of the blanks. Each blank can only include one statement or expression.

Click here to open the diagram in a new window/tab




```
def v(o, t, e):
    def m(y):

        _________
           (a)

        _________
           (b)

    def n(o):

        o.append(_________)
```

```
                    (c)

        o.append(_________)
                    (d)

    m(e)
    n([t])
    e = 2

m = [3, 4]

v(m, 5, 6)
```

(a) **(2 pt)** Fill in blank (a). **You may not write any numbers or arithmetic operators (+, -, *, /, //, **) in your solution.**

nonlocal o

(b) **(2 pt)** Fill in blank (b). **You may not write any numbers or arithmetic operators (+, -, *, /, //, **) in your solution.**

o = [o, t]

(c) **(2 pt)** Fill in blank (c). **You may not write any numbers or arithmetic operators (+, -, *, /, //, **) in your solution.**

[e]

(d) **(2 pt)** Which of these could fill in blank (d)? **Check all that apply.**

- ☐ `o`
- ☐ `[o]`
- ■ `list(o)`
- ☐ `[list(o)]`
- ☐ `list([o])`
- ■ `o + []`
- ■ `[o[0], o[1]]`
- ■ `o[:]`

2. (10 points) Yield, Fibonacci!

(a) (4 points)

Implement `fibs`, a generator function that takes a one-argument pure function `f` and yields all Fibonacci numbers `x` for which `f(x)` returns a true value.

The Fibonacci numbers begin with 0 and then 1. Each subsequent Fibonacci number is the sum of the previous two. Yield the Fibonacci numbers in order.

```
def fibs(f):
    """Yield all Fibonacci numbers x for which f(x) is a true value.

    >>> odds = fibs(lambda x: x % 2 == 1)
    >>> [next(odds) for i in range(10)]
    [1, 1, 3, 5, 13, 21, 55, 89, 233, 377]
    >>> bigs = fibs(lambda x: x > 20)
    >>> [next(bigs) for i in range(10)]
    [21, 34, 55, 89, 144, 233, 377, 610, 987, 1597]
    >>> evens = fibs(lambda x: x % 2 == 0)
    >>> [next(evens) for i in range(10)]
    [0, 2, 8, 34, 144, 610, 2584, 10946, 46368, 196418]
    """

    n, m = 0, 1

    while _________:
              (a)

        if _________:
                (b)

            _________
               (c)

        _________
           (d)
```

i. (1 pt) Which of these could fill in blank (a)?

- ◯ `f(n)`
- ◯ `f(m)`
- ◯ `f(n) or f(m)`
- ◯ `f(n) and f(m)`
- ● `True`
- ◯ `False`

ii. (1 pt) Which of these could fill in blank (b)?

- ● `f(n)`
- ○ `f(m)`
- ○ `f(n) or f(m)`
- ○ `f(n) and f(m)`
- ○ `True`
- ○ `False`

iii. (1 pt) Fill in blank (c).

yield n

iv. (1 pt) Fill in blank (d).

n, m = m, n + m

(b) (6 points)

Definition. For a linked list `s`, the *index* of an element is the number of times `rest` appears in the smallest dot expression containing only `s`, `rest`, and `first` that evaluates to that element. For example, in the linked list `s = Link(5, Link(7, Link(9, Link(11))))`,

- The index of 5 (`s.first`) is 0.
- The index of 7 (`s.rest.first`) is 1.
- The index of 11 (`s.rest.rest.rest.first`) is 3.

Implement `filter_index`, a function that takes a one-argument pure function `f` and a `Link` instance `s`. It returns a `Link` containing all elements of `s` that have an **index** `i` for which `f(i)` returns a true value.

Assume that `s` is a finite linked list of numbers that contains no repeated elements. The `Link` class appears on Page 2 (left column) of the Midterm 2 Study Guide.

```
def filter_index(f, s):
    """Return a Link containing the elements of Link s that have an index i for
    which f(i) is a true value.

    >>> powers = Link(1, Link(2, Link(4, Link(8, Link(16, Link(32))))))
    >>> filter_index(lambda x: x < 4, powers)
    Link(1, Link(2, Link(4, Link(8))))
    >>> filter_index(lambda x: x % 2 == 1, powers)
    Link(2, Link(8, Link(32)))
    """

    def helper(i, s):

        if s is Link.empty:

            return s

        filtered_rest = _________
                           (a)
        if _________:
              (b)

            return _________
                       (c)

        else:

            return filtered_rest

    return _________
              (d)
```

i. (1 pt) Which of these could fill in blank (a)?

- ● `helper(i + 1, s.rest)`
- ○ `helper(i + 1, s.rest.rest)`
- ○ `filter_index(f, s.rest)`
- ○ `filter_index(f, s.rest.rest)`
- ○ `Link(helper(i + 1, s.rest))`
- ○ `Link(helper(i + 1, s.rest.rest))`
- ○ `Link(filter_index(f, s.rest))`
- ○ `Link(filter_index(f, s.rest.rest))`

ii. (1 pt) Fill in blank (b).

f(i)

iii. (2 pt) Fill in blank (c).

Link(s.first, filtered_rest)

iv. (2 pt) Fill in blank (d).

helper(0, s)

3. (12 points) Sparse Lists

The `most_common` function returns the most common element in a non-empty list. You do not need to implement this function. Assume that it is implemented for you.

```
def most_common(s):
    """Return the most common element in non-empty list s. In case of a tie,
    return the most common element that appears first in s.

    >>> most_common([3, 1, 4, 1, 5, 9])
    1
    >>> most_common([3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5])
    5
    >>> most_common([2, 7, 1, 8, 2, 8, 1, 8, 2, 8])
    8
    >>> most_common([3, 5, 7, 7, 7, 5, 5])
    5
    >>> most_common([3, 7, 5, 5, 7, 7])
    7
    """
```

Implement the `SparseList` class. A `SparseList` instance represents a non-empty list `s`.

- Its `common` attribute is the most common value in `s`.
- Its `others` dictionary has a value for every element in `s` that is not `common`. The corresponding key is the **index** for that value in `s`.
- Its `item` method takes a non-negative integer `i` and returns `s[i]` or the string `'out of range'` if `i` is not smaller than the length of `s`.
- Its `items` method returns a list with the same elements as `s` in the same order as `s`.

```
class SparseList:
    """Represent a non-empty list as a most common value and a dictionary from
    indices to values that contains only values that are not the most common.

    >>> pi = SparseList([3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5])
    >>> pi.common
    5
    >>> pi.others
    {0: 3, 1: 1, 2: 4, 3: 1, 5: 9, 6: 2, 7: 6, 9: 3}
    >>> [pi.item(0), pi.item(1), pi.item(2), pi.item(3), pi.item(4)]
    [3, 1, 4, 1, 5]
    >>> pi.item(10)
    5
    >>> pi.item(11)
    'out of range'
    >>> pi.items()
    [3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5]
    """
    def __init__(self, s):
        ...
    def item(self, i):
        ...
    def items(self):
        ...
```

(a) (5 points)

Implement the `__init__` method, which takes a list `s`.

```
def __init__(self, s):

    assert s, 's cannot be empty'

    self.n = len(s)

    self.common = most_common(_________)
                                 (a)

    self.others = { _________: _________ for i in range(_________) if _________ }
                       (b)        (c)                      (d)           (e)
```

i. (1 pt) Fill in blank (a).

s

ii. (1 pt) Which of these could fill in blank (b)?

- ● `i`
- ○ `self.i`
- ○ `n`
- ○ `self.n`
- ○ `s[i]`
- ○ `s`

iii. (1 pt) Fill in blank (c).

s[i]

iv. (1 pt) Which of these could fill in blank (d)? **Check all that apply.**

- ☐ `s`
- ■ `len(s)`
- ☐ `self.s`
- ☐ `len(self.s)`
- ☐ `n`
- ☐ `len(n)`
- ■ `self.n`
- ☐ `len(self.n)`

v. (1 pt) Fill in blank (e).

s[i] != self.common

(b) (3 points)

Implement the `item` method, which takes a non-negative integer `i`.

```
def item(self, i):
    """Return s[i] or 'out of range' if i is not smaller than the length of s."""

    assert i >= 0, 'index i must be non-negative'

    if _________:
          (a)

        return 'out of range'

    elif _________:
            (b)

        return _________
                  (c)

    else:

        return self.common
```

i. (1 pt) Fill in blank (a).

i >= self.n

ii. (1 pt) Fill in blank (b).

i in self.others

iii. (1 pt) Which of these could fill in blank (c)?

- ○ `others[i]`
- ● `self.others[i]`
- ○ `others[self.i]`
- ○ `self.others[self.i]`
- ○ `others`
- ○ `self.others`

(c) (4 points)

Implement the `items` method.

```
def items(self):

    """Return a list with the same elements as s in the same order as s."""

    return [_________ for i in _________]
                (a)                (b)
```

i. (2 pt) Fill in blank (a). You may not use `and`, `or`, `if`, `else`, `[`, `]`, or `get`.

Hint: Don't repeat yourself.

self.item(i)

ii. (2 pt) Fill in blank (b).

range(self.n)

4. (20 points) Fork It

The *tree* data abstraction, which is implemented by the constructor `tree`, selectors `branches` and `label`, and helper functions `is_leaf` and `is_tree` appear on Page 2 (left column) of the Midterm 2 Study Guide. You may call these functions. Do not violate the abstraction barriers of the tree data abstraction.

(a) (4 points)

Implement `max_path`, which takes a tree `t` whose labels are **all positive** numbers and returns the largest sum of the labels along a path from the root of `t` to one of its leaves.

You may call `tree`, `label`, `branches`, `is_leaf`, `is_tree`, and `max_path`.

```
def max_path(t):
    """Return the largest sum of labels along any path from the root to a leaf
    of tree t, which has positive numbers as labels.

    >>> a = tree(1, [tree(2), tree(3), tree(4, [tree(5)])])
    >>> max_path(a) # 1 + 4 + 5
    10
    >>> b = tree(6, [a, a, a])
    >>> max_path(b) # 6 + 1 + 4 + 5
    16
    """

    return _________ + max(_________ + _________)
              (a)             (b)         (c)
```

i. (1 pt) Which of the following could fill in blank (a)?

○ `t`

● `label(t)`

○ `[t]`

○ `[label(t)]`

○ `[0]`

○ `sum([b for b in branches(t)])`

○ `sum([label(b) for b in branches(t)])`

ii. (1 pt) Which of the following could fill in blank (b)?

○ `t`

○ `label(t)`

○ `[t]`

○ `[label(t)]`

○ `[0]` CORRECT

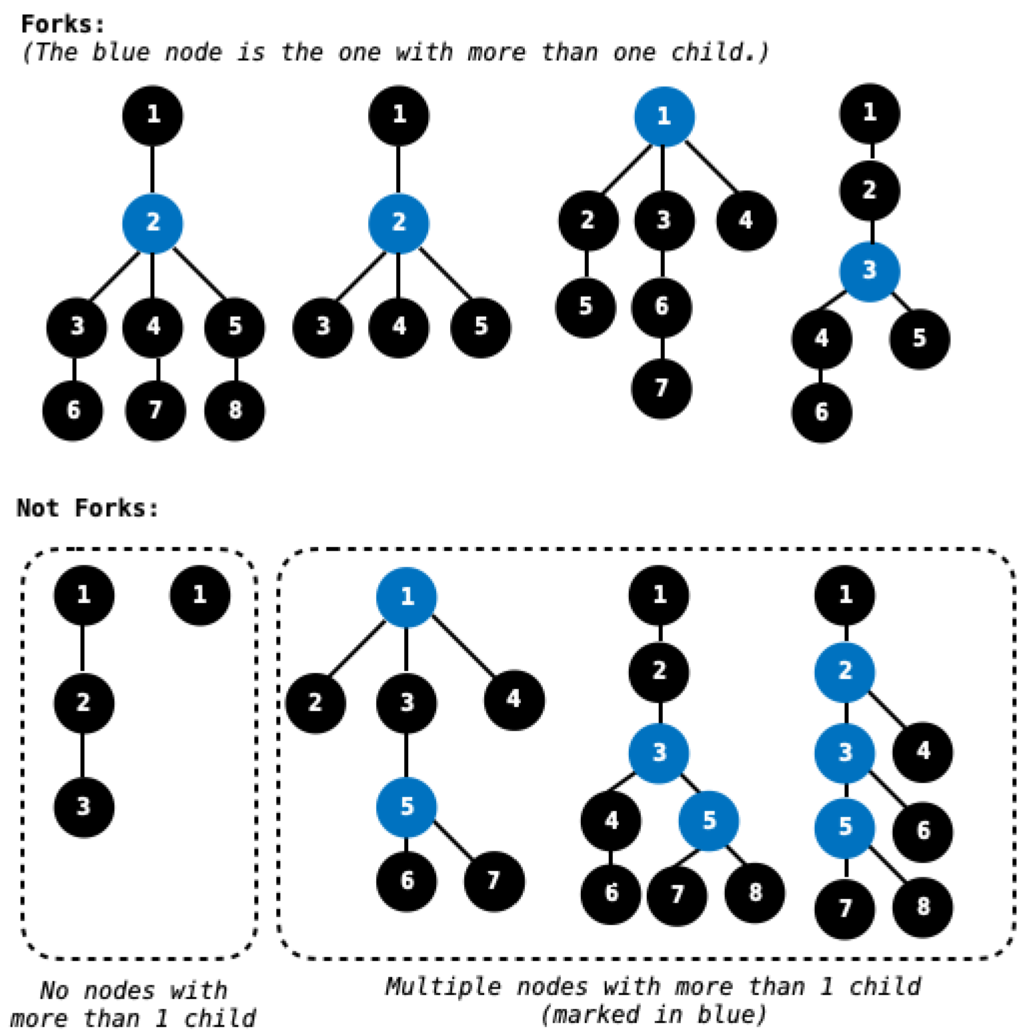
○ `[label(b) for b in branches(t)]`

iii. (2 pt) Fill in blank (c). You may not use the word `default`.

[max_path(b) for b in branches(t)]

(b) (8 points)

Definition. A *fork* is a tree in which **exactly one node** has more than one child.



Implement `is_fork` and its helper function `slide`. The `is_fork` function takes a *tree* `t` and returns `True` if `t` is a fork and `False` otherwise.

You may call `tree`, `label`, `branches`, `is_leaf`, `is_tree`, `max_path`, `is_fork`, and `slide`.

```
def is_fork(t):
    """Return whether tree t is a fork.

    >>> is_fork(tree(1, [tree(2, [tree(3), tree(4), tree(5)])]))
    True
    >>> is_fork(tree(1, [tree(2, [tree(3)]), tree(4)]))
    True
    >>> is_fork(tree(1, [tree(2), tree(3), tree(4)]))
    True
    >>> is_fork(tree(1, [tree(2, [tree(3, [tree(5)]), tree(4, [tree(6)])])]))
    True
    >>> is_fork(tree(1))
    False
    >>> is_fork(tree(1, [tree(2, [tree(3)])]))
    False
```

```
    >>> is_fork(tree(1, [tree(2, [tree(3)]), tree(4, [tree(5), tree(6)])]))
    False
    >>> is_fork(tree(1, [tree(2, [tree(3, [tree(5, [tree(7), tree(8)]), tree(6)]), tree(4)])]))
    False
    """

    neck = slide(t)

    if is_leaf(neck):

        _________
           (a)

    return _________([_________ for b in branches(_________)])
              (b)        (c)                          (d)

def slide(t):
    """Return the deepest node within tree t whose ancestors all have exactly one child.

    Definition: The ancestors of a node include its parent and the parents of all its ancestors.

    >>> deepest = slide(tree(1, [tree(2, [tree(3)])]))
    >>> label(deepest)
    3
    >>> label(slide(tree(1, [tree(2, [tree(3), tree(4)])])))
    2
    """

    while _________:
             (e)

        t = _________
               (f)

    return t
```

i. **(1 pt)** Fill in blank (a).

return False

ii. **(1 pt)** Which of the following could fill in blank (b)?

- ● `all`
- ○ `any`
- ○ `list`
- ○ `tree`
- ○ `branches`
- ○ `label`

iii. (2 pt) Fill in blank (c).

is_leaf(slide(b))

iv. (1 pt) Which of the following could fill in blank (d)?

- ○ `t`
- ○ `tree`
- ● `neck`
- ○ `label(t)`
- ○ `label(tree)`
- ○ `label(neck)`

v. (1 pt) Which of the following could fill in blank (e)?

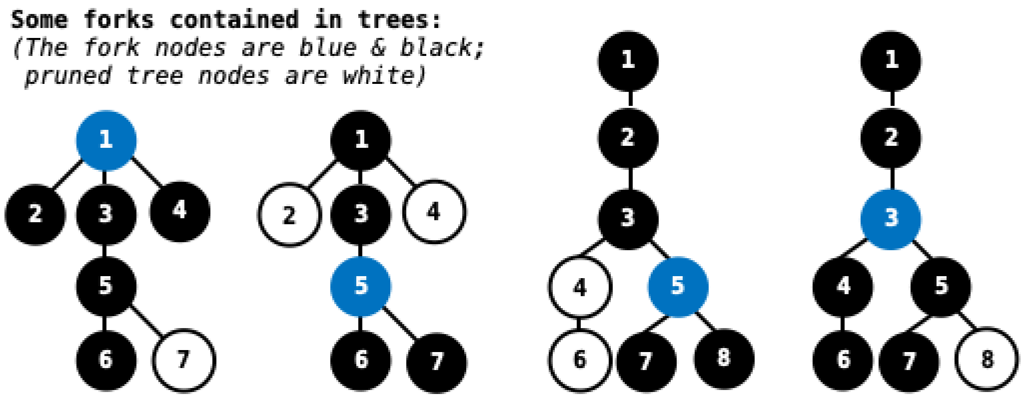
- ● `len(branches(t)) == 1`
- ○ `len(branches(t)) > 1`
- ○ `len(branches(t)) != 1`
- ○ `branches(t) == 1`
- ○ `branches(t) > 1`
- ○ `branches(t) != 1`

vi. (2 pt) Fill in blank (f).

branches(t)[0]

(c) (8 points)

Definition. A tree `t` *contains* a fork `u` if `u` is the result of pruning zero or more nodes from `t`.



Implement `max_fork`, which takes a tree `t` whose labels are all positive integers. It returns the largest sum of the labels of a fork that is contained in `t`. If `t` does not contain any forks, then `max_fork` returns 0.

You may call `tree`, `label`, `branches`, `is_leaf`, `is_tree`, `max_path`, `is_fork`, `slide`, and `max_fork`.

```
def max_fork(t):
    """Return the largest sum of the labels in any fork contained in tree t,
    which has positive numbers as labels. If t contains no forks, return 0.


    >>> a = tree(1, [tree(2), tree(3), tree(4, [tree(5)])])
    >>> max_fork(a) # 1 + 2 + 3 + 4 + 5
    15
    >>> b = tree(6, [a, a, a])
    >>> max_fork(b)  #         6 + (1 + 4 + 5) + (1 + 4 + 5) + (1 + 4 + 5)
    36
    >>> c = tree(7, [tree(8), b, tree(9)])
    >>> max_fork(c)  #     7 + (6 + (1 + 4 + 5) + (1 + 4 + 5) + (1 + 4 + 5))
    43
    >>> d = tree(9, [c])
    >>> max_fork(d)  # 9 + 7 + (6 + (1 + 4 + 5) + (1 + 4 + 5) + (1 + 4 + 5))
    52
    >>> max_fork(tree(1, [tree(2, [tree(3)])])) # No forks here!
    0
    """

    n = len(branches(t))

    if n == 0:

        return 0

    elif n == 1:

        below = _________
                   (a)

        if _________:
              (b)
```

```
            return _________ + below
                      (c)

        else:

            return 0

    else:

        here = sum([_________ for b in branches(t)])
                      (d)

        there = max([_________ for b in branches(t)])
                       (e)

        return label(t) + max(here, there)
```

i. (2 pt) Fill in blank (a).

max_fork(branches(t)[0])

ii. (1 pt) Which of the these could fill in blank (b)?

- ● `below > 0`
- ○ `label(t) > 0`
- ○ `max_fork(t) > 0`
- ○ `True`
- ○ `max_fork(t) > max_path(t)`
- ○ `is_fork(t)`

iii. (1 pt) Which of these could fill in blank (c)?

- ○ `1`
- ● `label(t)`
- ○ `max_path(t)`
- ○ `label(slide(t))`
- ○ `max([label(b) for b in branches(t)])`
- ○ `label(branches(t)[0])`

iv. (2 pt) Fill in blank (d).

max_path(b)

v. (2 pt) Fill in blank (e).

max_fork(b)

No more questions.