CS 61A Structure and Interpretation of Computer Programs
Spring 2017

Test 2 (revised)

INSTRUCTIONS

- You have 2 hours to complete the exam.
- The exam is open book, open notes, closed computer, closed calculator.
- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.

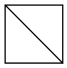
Last name	
First name	
Student ID number	
CalCentral email (`@berkeley.edu`)	
TA	
Name of the person to your left	
Name of the person to your right	
Room in which you are taking exam	
Seat number in the exam room	
I pledge my honor that during this examination I have neither given nor received assistance. **(please sign)**	

Reference Material.

```
# Linked Lists

class Link:
    """A linked list cell.
    >>> L = Link(0, Link(1))
    >>> L.first
`   0
    >>> L.rest
    Link(1)
    >>> L.first = 2
    >>> L
    Link(2, Link(1))
    >>> L.rest = Link.empty
    >>> L
    Link(2)
    """
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest is Link.empty:
             return "Link({})".format(self.first)
        else:
             return "Link({}, {})".format(self.first, self.rest)

# Trees

class Tree:
    """A tree node."""

    def __init__(self, label, branches=[]):
        for c in branches:
            assert isinstance(c, Tree)
        self.label = label
        self.branches = branches

    def is_leaf(self):
        return not self.branches
```

1. (12 points) Pointers

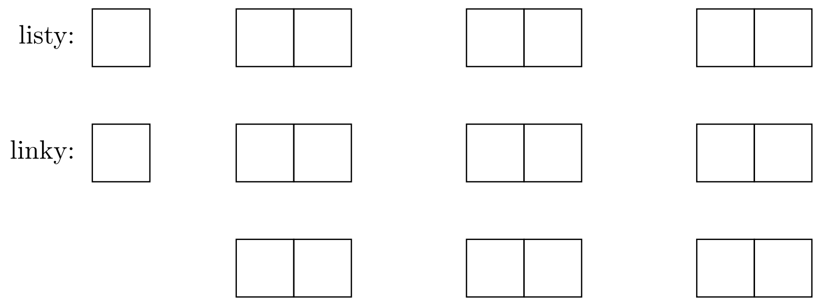
In the following problems, single boxes are variables that contain pointers, and double boxes are `Links` (see the definition of `Link` on page **??**). To show that a box contains a pointer to the empty list, draw the box like this:



In parts (a) and (b), add arrows and values to the object skeletons on the right to show the final state of the program. Not all boxes will be used. (For examples of what kinds of box and pointer diagrams we're looking for, you might look at parts (c) and (d) first.)

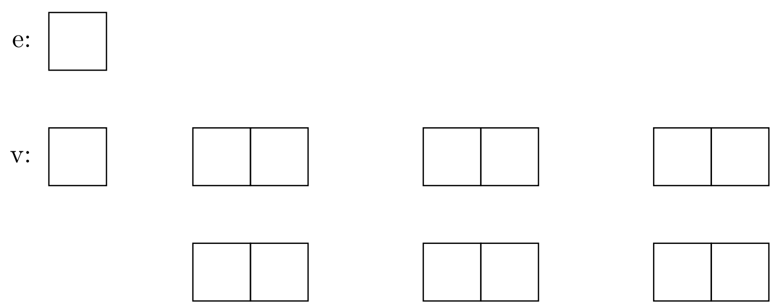
(a) (3 pt)

```
listy = Link(0, Link(1))

def nest(L):
    if L is Link.empty:
        return L
    N = nest(L.rest)
    L.first = Link(L.first, N)
    return L.first

linky = nest(listy)
```

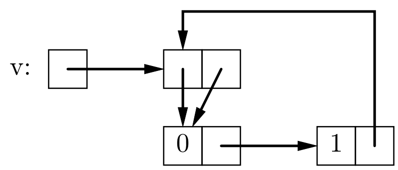


(b) (3 pt)

```
v = Link(0, Link(1, Link(2)))
e = v.rest.rest
e.rest = v.rest
v.rest.rest = v
v.rest = Link.empty
```

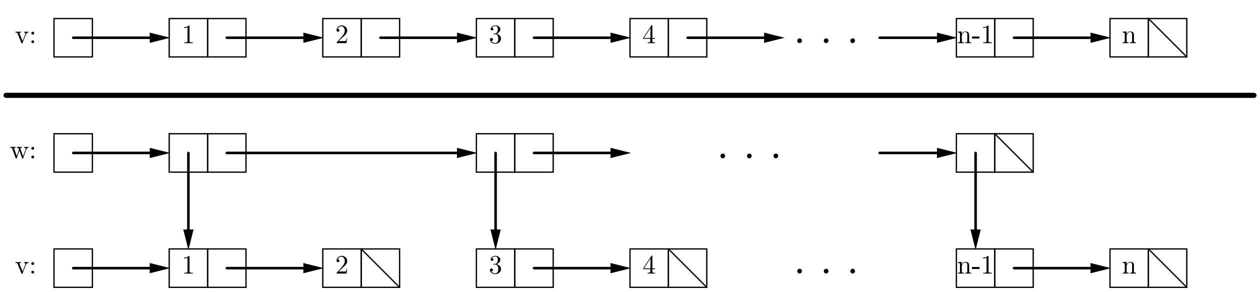


(c) **(3 pt)** Show Python code that will produce the situation shown in the diagram. (An arrow pointing to a `Link` may be shown as pointing anywhere on the double box for that `Link`.)




```
v = Link(________________________________________________________________)


v._________________________ = _______________________________________________


v._________________________ = _______________________________________________
```

(d) **(3 pt)** Show Python code that converts the situation shown above the line into that shown below the line. Assume n is even.




```
def split2(L):
    """Assuming that linked list L has even length, breaks L into
    two-element linked lists, and returns a linked list of those lists."""


    if ____________________________:

           return _________________________
    else:

           result = Link(L, _________________________________________________)

           ______________________________ = Link.empty

           return result

w = split2(v)
```

2. (6 points) Complexity

(a) (1.5 pt) Indicate which of the following assertions are true by circling the letters for those statements. An assertion such as $\Theta(f(n)) \subseteq \Theta(g(n))$ means "any function that is in $\Theta(f(n))$ is also in $\Theta(g(n))$," and $\Theta(f(n)) = \Theta(g(n))$ if and only if $\Theta(f(n)) \subseteq \Theta(g(n))$ and $\Theta(g(n)) \subseteq \Theta(f(n))$.

A. If $f(n) \in \Theta(1)$ and $g(n) \in \Theta(1)$, then $\Theta(|f(n)| + |g(n)|) \in \Theta(1)$.

B. If $\Theta(f(n)) = \Theta(g(n))$, and $g(n) > 0$ everywhere, then $f(n)/g(n) \in \Theta(1)$.

C. $\Theta(x^2) \subseteq \Theta(x^3)$.

D. $\Theta(2^x) \subseteq \Theta(2^x + x^2)$.

E. If $f(n) \in \Theta(1000 \cdot x^3)$, then $f(20) > 800$.

(b) (1.5 pt) Consider the function

```
def num_kinks(L):
    c = 0
    i = 0
    while i < len(L):
        j = i
        while j < len(L):
            while j < len(L):
                if kink(L[i], L[j]):
                    c += 1
                    break
                j += 1
            j += 1
        i += 1
    return c
```

Circle the order of growth that best describes the worst-case execution time (measured by the number of calls to `kink`) of a call to `num_kinks` as a function of N, the length of `L`.

A. $\Theta(\log N)$

B. $\Theta(N)$

C. $\Theta(N^2)$

D. $\Theta(N^3)$

E. $\Theta(2^N)$

(c) (1.5 pt) Consider the following function on `Trees`

```
def count_subtrees(T, p):
    if p(T.label):
        return 1
    else:
        return sum([count_subtrees(child, p) for child in T.branches])
```

Assuming that the maximum number of children of any node is 3, circle the order of growth that best describes the worst-case execution time (measured by the number of calls to `p`) of a call to `count_subtrees` as a function of N, the number of nodes in `T`.

A. $\Theta(\log N)$

B. $\Theta(N)$

C. $\Theta(N^2)$

D. $\Theta(N^3)$

E. $\Theta(3^N)$

(d) (1.5 pt) For the same function as in part (c) above, and again assuming that the maximum number of children of any node is 3, circle the order of growth that best describes the worst-case execution time (measured by the number of calls to `p`) of a call to `count_subtrees` as a function of H, the height of `T`.

A. $\Theta(\log H)$

B. $\Theta(H)$

C. $\Theta(H^2)$

D. $\Theta(H^3)$

E. $\Theta(3^H)$.

3. (1 points) From the Sum of Human Knowledge

This Renaissance composer, famous for his harmonically innovative madrigals, was also infamous for murdering his wife and her lover and thereafter having himself beaten regularly by one of his servants. Who was he?

4. (8 points) OOPs!

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. If an error occurs, write "`ERROR`". No answer requires more than 3 lines. (It's possible that all of them require even fewer.)

```
class Thing:
    id = 0

    def fidget(self, n):
        print(n, "A", self.id)

    def fuss(self, x, n):
        print(n, "B")
        self.fidget(n)
        x.fidget(n)

    def twitch(self, n):
        self.waffle(n)

class Gadget(Thing):
    id = 1

class Whatsit(Gadget):
    def fidget(self, n):
        print(n, "D", self.id)

    def waffle(self, n):
        print(n, "D")

    def fiddle(self, x, n):
        x.waffle(n)

t1 = Thing()
t2 = Gadget()
t3 = Whatsit()
t4 = Whatsit()
t3.id = 2
```

Expression	**Interactive Output**
`t3.fidget(1)`	
`t4.fidget(2)`	
`t4.fuss(t1, 3)`	
`t4.fiddle(t4, 4)`	
`t4.fiddle(t1, 5)`	
`Thing.id = 3` `t1.fidget(6)`	

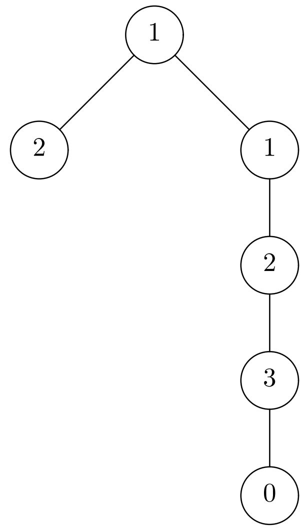
5. (8 points) Inflections

Fill in the definition of class `Wrinkles` to conform to its doc comment. You need not use all the lines shown.

```
class Wrinkles:
    """An object that contains a sequence of items and a predicate (true/false
    function) and that, when iterated over, produces adjacent pairs of items
    in the sequence that satisfy the predicate.
    >>> w = Wrinkles([1, 2, 3, 2, 4, 8, 5, 4], lambda x, y: x > y)
    >>> for p in w:
    ...     print(p)
    (3, 2)
    (8, 5)
    (5, 4)
    """

    def __init__(self, L, wrinkle):
        self._L = L
        self._wrinkle = wrinkle


    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________

    ___________________________________________________________________________
```

6. (8 points) Tree Paths

Given a tree, t, find the length of the longest downward sequence of node labels in the tree that are increasing consecutive integers. For example, in this tree, the longest such sequence has three labels (1, 2, 3):



As illustrated, the longest sequence can start and end anywhere in the tree, not just the root. (**Hint:** don't forget there's a `max` function.) [The original skeleton was flawed. The original skeleton appears here, and a revised skeleton on the next page.]

```
def longest_seq(t):
    """The length of the longest downward sequence of nodes in T whose
    labels are consecutive integers.
    >>> t = Tree(1, [Tree(2), Tree(1, [Tree(2, [Tree(3, [Tree(0)])])])])
    >>> longest_seq(t) # 1 -> 2 -> 3
    3
    >>> t = Tree(1)
    >>> longest_seq(t)
    1
    """

    if ____________________________________________________________:

        return ____________________________________________________

    max_len = _____________________________________________________

    for ___________________________________________________________:

        if ________________________________________________________:

            _______________________________________________________
        else:

            _______________________________________________________

    return max_len
```

Here is the corrected skeleton.

```
def longest_seq(t):
    """The length of the longest downward sequence of nodes in T whose
    labels are consecutive integers.
    >>> t = Tree(1, [Tree(2), Tree(1, [Tree(2, [Tree(3, [Tree(0)])])])])
    >>> longest_seq(t) # 1 -> 2 -> 3
    3
    >>> t = Tree(1)
    >>> longest_seq(t)
    1
    """
    max_len = 1

    def longest(t):
        """Returns longest downward sequence of nodes starting at T whose
        labels are consecutive integers.  Updates max_len to that length,
        if greater."""

        _________________________________________________________________

        n = 1
        if ________________________________________________________________:

            for ________________________________________________________

                ________________________________________________________

                if ______________________________________________________

                    n = ________________________________________________

            max_len = __________________________________________________
        return n
    longest(tr)
    return max_len
```

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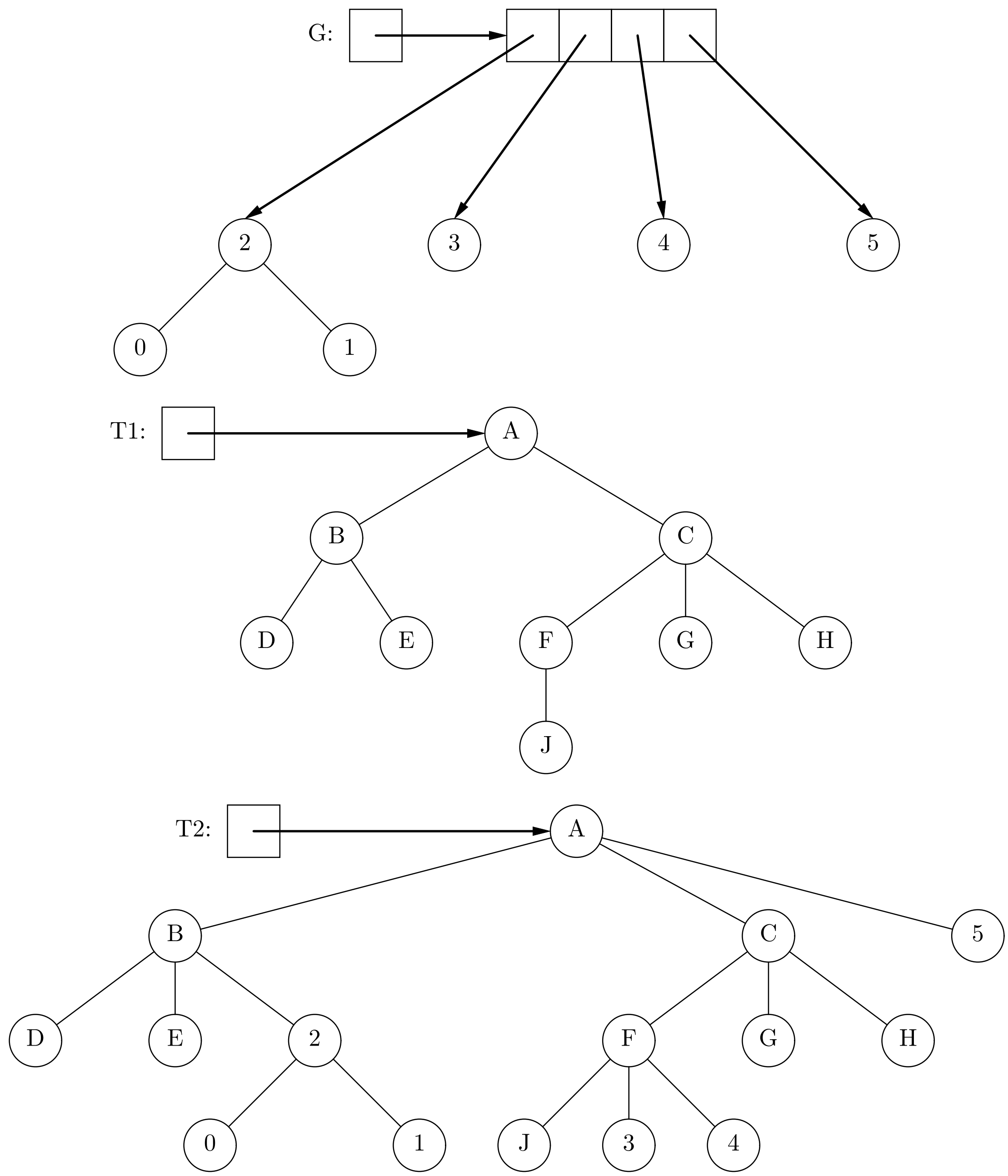
7. (8 points) Grafting

We want to insert ("graft") branches from a sequence of trees onto a tree in places where a non-leaf node has fewer than K branches, where K is a parameter. For example, given the list of four trees `G` created by

```
G = [ Tree(2, [Tree(0), Tree(1)]), Tree(3), Tree(4), Tree(5) ]
```

and the tree `T1` shown below, we want `T2 = graft(T1, G, 3)` to destructively (and without creating any new tree nodes) turn `T1` into the tree `T2`:



The list of trees (`G` in the example above) will always have enough items to fill all necessary places. Trees are inserted in postorder (that is, bottom to top, left to right).

Fill in the `graft` function here. You need not use all the lines.

```
def graft(T, L, k):
    """Returns the tree created by destructively adding trees from L to
    non-leaf nodes of T with fewer than K branches.  Assume that L has enough
    items to fill all necessary places.  Fill in trees in postorder (bottom to
    top, left to right)."""

    grafts = iter(L)   # Don't have to use this, but it may be useful.

    def do_grafts(tr):


        if __________________________________________________________________:


            for ______________________________________________________________:


                ______________________________________________________________


            while ____________________________ < ____________________________:


                ______________________________________________________________


                ______________________________________________________________

    do_grafts(T)

    return T
```