INSTRUCTIONS

- You have 2 hours to complete the exam.
- The exam is open book, open notes, closed computer, closed calculator.
- Mark your answers on the exam itself. We will not grade answers written on scratch paper.
# Linked Lists

class Link:
    """A linked list cell."
    >>> L = Link(0, Link(1))
    >>> L.first
    0
    >>> L.rest
    Link(1)
    >>> L.first = 2
    >>> L
    Link(2, Link(1))
    >>> L.rest = Link.empty
    >>> L
    Link(2)
    """
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

def __repr__(self):
    if self.rest is Link.empty:
        return "Link({})".format(self.first)
    else:
        return "Link({}, {})".format(self.first, self.rest)

# Trees

class Tree:
    """A tree node."
    
    def __init__(self, label, branches=[]):
        for c in branches:
            assert isinstance(c, Tree)
            self.label = label
            self.branches = branches

    def is_leaf(self):
        return not self.branches
1. (12 points) **Pointers** *(All are in Scope: Environment Diagram, Linked Lists)*

In the following problems, single boxes are variables that contain pointers, and double boxes are Links (see the definition of Link on page 2). To show that a box contains a pointer to the empty list, draw the box like this:

In parts (a) and (b), add arrows and values to the object skeletons on the right to show the final state of the program. Not all boxes will be used. (For examples of what kinds of box and pointer diagrams we’re looking for, you might look at parts (c) and (d) first.)

(a) (3 pt)

```python
listy = Link(0, Link(1))
def nest(L):
    if L is Link.empty:
        return L
    N = nest(L.rest)
    L.first = Link(L.first, N)
    return L.first
linky = nest(listy)
```

(b) (3 pt)

```python
v = Link(0, Link(1, Link(2)))
e = v.rest.rest
e.rest = v.rest
v.rest.rest = v
v.rest = Link.empty
```
(c) (3 pt) Show Python code that will produce the situation shown in the diagram. (An arrow pointing to a Link may be shown as pointing anywhere on the double box for that Link.)

```
v: 0 1
```

```
v = Link(__________________________________________)
```

```
v.________________________ = ______________________________________________
```

```
v.________________________ = ______________________________________________
```

(d) (3 pt) Show Python code that converts the situation shown above the line into that shown below the line. Assume \( n \) is even.

```
v: 1 2 3 4 . . . n-1 n
```

```
w: ...
```

```
v: 1 2 3 4 . . . n-1 n
```

```
def split2(L):
    """Assuming that linked list L has even length, breaks L into two-element linked lists, and returns a linked list of those lists."""
    if __________________________:
        return ________________________
    else:
        result = Link(L, _________________________________________________)
        ________________________ = Link.empty
        return result
    w = split2(v)
```
2. (6 points) Complexity (At least one of these is out of Scope: Growth)

(a) (1.5 pt) Indicate which of the following assertions are true by circling the letters for those statements. An assertion such as $\Theta(f(n)) \subseteq \Theta(g(n))$ means “any function that is in $\Theta(f(n))$ is also in $\Theta(g(n))$,” and $\Theta(f(n)) = \Theta(g(n))$ if and only if $\Theta(f(n)) \subseteq \Theta(g(n))$ and $\Theta(g(n)) \subseteq \Theta(f(n))$.

A. If $f(n) \in \Theta(1)$ and $g(n) \in \Theta(1)$, then $\Theta(|f(n)| + |g(n)|) \in \Theta(1)$.

B. If $\Theta(f(n)) = \Theta(g(n))$, and $g(n) > 0$ everywhere, then $f(n)/g(n) \in \Theta(1)$.

C. $\Theta(x^2) \subseteq \Theta(x^3)$.

D. $\Theta(2^x) \subseteq \Theta(2^x + x^2)$.

E. If $f(n) \in \Theta(1000 \cdot x^3)$, then $f(20) > 800$.

(b) (1.5 pt) Consider the function

```python
def num_kinks(L):
    c = 0
    i = 0
    while i < len(L):
        j = i
        while j < len(L):
            while j < len(L):
                if kink(L[i], L[j]):
                    c += 1
                    break
            j += 1
        j += 1
        i += 1
    return c
```

Circle the order of growth that best describes the worst-case execution time (measured by the number of calls to kink) of a call to num_kinks as a function of $N$, the length of $L$.

A. $\Theta(\log N)$

B. $\Theta(N)$

C. $\Theta(N^2)$

D. $\Theta(N^3)$

E. $\Theta(2^N)$
(c) (1.5 pt) Consider the following function on Trees

```python
def count_subtrees(T, p):
    if p(T.label):
        return 1
    else:
        return sum([count_subtrees(child, p) for child in T.branches])
```

Assuming that the maximum number of children of any node is 3, circle the order of growth that best describes the worst-case execution time (measured by the number of calls to \( p \)) of a call to `count_subtrees` as a function of \( N \), the number of nodes in \( T \).

A. \( \Theta(\log N) \)
B. \( \Theta(N) \)
C. \( \Theta(N^2) \)
D. \( \Theta(N^3) \)
E. \( \Theta(3^N) \)

(d) (1.5 pt) For the same function as in part (c) above, and again assuming that the maximum number of children of any node is 3, circle the order of growth that best describes the worst-case execution time (measured by the number of calls to \( p \)) of a call to `count_subtrees` as a function of \( H \), the height of \( T \).

A. \( \Theta(\log H) \)
B. \( \Theta(H) \)
C. \( \Theta(H^2) \)
D. \( \Theta(H^3) \)
E. \( \Theta(3^H) \).

3. (1 points) From the Sum of Human Knowledge

This Renaissance composer, famous for his harmonically innovative madrigals, was also infamous for murdering his wife and her lover and thereafter having himself beaten regularly by one of his servants. Who was he?
4. (8 points) OOPs! (All are in Scope: Environment Diagram, Objects)

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. If an error occurs, write "ERROR". No answer requires more than 3 lines. (It’s possible that all of them require even fewer.)

class Thing:
    id = 0

    def fidget(self, n):
        print(n, "A", self.id)

    def fuss(self, x, n):
        print(n, "B")
        self.fidget(n)
        x.fidget(n)

    def twitch(self, n):
        self.waffle(n)

class Gadget(Thing):
    id = 1

class Whatsit(Gadget):
    def fidget(self, n):
        print(n, "D", self.id)

    def waffle(self, n):
        print(n, "D")

    def fiddle(self, x, n):
        x.waffle(n)

t1 = Thing()
t2 = Gadget()
t3 = Whatsit()
t4 = Whatsit()
t3.id = 2

<table>
<thead>
<tr>
<th>Expression</th>
<th>Interactive Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>t3.fidget(1)</td>
<td></td>
</tr>
<tr>
<td>t4.fidget(2)</td>
<td></td>
</tr>
<tr>
<td>t4.fuss(t1, 3)</td>
<td></td>
</tr>
<tr>
<td>t4.fiddle(t4, 4)</td>
<td></td>
</tr>
<tr>
<td>t4.fiddle(t1, 5)</td>
<td></td>
</tr>
<tr>
<td>Thing.id = 3</td>
<td></td>
</tr>
<tr>
<td>t1.fidget(6)</td>
<td></td>
</tr>
</tbody>
</table>
5. (8 points) Inflections (All are in Scope: OOP, Lists, Iterators and Generators)

Fill in the definition of class Wrinkles to conform to its doc comment. You need not use all the lines shown.

```python
class Wrinkles:
    """An object that contains a sequence of items and a predicate (true/false function) and that, when iterated over, produces adjacent pairs of items in the sequence that satisfy the predicate.
    >>> w = Wrinkles([1, 2, 3, 2, 4, 8, 5, 4], lambda x, y: x > y)
    >>> for p in w:
    ...    print(p)
    (3, 2)
    (8, 5)
    (5, 4)
    """

    def __init__(self, L, wrinkle):
        self._L = L
        self._wrinkle = wrinkle
```

...
6. (8 points) Tree Paths (All are in Scope: Trees, Nonlocal)

Given a tree, t, find the length of the longest downward sequence of node labels in the tree that are increasing consecutive integers. For example, in this tree, the longest such sequence has three labels (1, 2, 3):

As illustrated, the longest sequence can start and end anywhere in the tree, not just the root. (Hint: don’t forget there’s a max function.) [The original skeleton was flawed. The original skeleton appears here, and a revised skeleton on the next page.]

def longest_seq(t):
    """The length of the longest downward sequence of nodes in T whose labels are consecutive integers.
    >>> t = Tree(1, [Tree(2), Tree(1, [Tree(2, [Tree(3, [Tree(0)])])])])
    >>> longest_seq(t) # 1 -> 2 -> 3
    3
    >>> t = Tree(1)
    >>> longest_seq(t)
    1
    """

    if ____________________________________________________________:
        return ____________________________________________________

    max_len = _____________________________________________________
    for ___________________________________________________________:
        if ________________________________________________________:
            ______________________________________________________
        else:
            ______________________________________________________

    return max_len
Here is the corrected skeleton.

def longest_seq(t):
    
    def longest(t):
        """Returns longest downward sequence of nodes starting at T whose labels are consecutive integers. Updates max_len to that length, if greater.""

        n = 1
        if __________________________________________________________________:
            for ______________________________________________________
                __________________________________________________________
                if _____________________________________________________
                    n = ________________________________________________
                    max_len = __________________________________________________
            return n
        longest(tr)
        return max_len
This page deliberately blank.
7. (8 points) Grafting (*All are in Scope: Trees, Lists, Iterators and Generators*)

We want to insert ("graft") branches from a sequence of trees onto a tree in places where a non-leaf node has fewer than \( K \) branches, where \( K \) is a parameter. For example, given the list of four trees \( G \) created by

\[
G = [ \text{Tree}(2, [\text{Tree}(0), \text{Tree}(1)]), \text{Tree}(3), \text{Tree}(4), \text{Tree}(5) ]
\]

and the tree \( T_1 \) shown below, we want \( T_2 = \text{graft}(T_1, G, 3) \) to destructively (and without creating any new tree nodes) turn \( T_1 \) into the tree \( T_2 \):

\[
\begin{array}{cccccc}
2 & 3 & 4 & 5 \\
0 & 1 \\
\end{array}
\]

\[
T_1: \quad A
\]

\[
\begin{array}{ccc}
B & C \\
D & E & F & G & H \\
J \\
\end{array}
\]

\[
T_2: \quad A
\]

\[
\begin{array}{ccc}
B & C & 5 \\
D & E & 2 & F & G & H \\
0 & 1 & J & 3 & 4 \\
\end{array}
\]

The list of trees (\( G \) in the example above) will always have enough items to fill all necessary places. Trees are inserted in postorder (that is, bottom to top, left to right).
Fill in the `graft` function here. You need not use all the lines.

```python
def graft(T, L, k):
    """Returns the tree created by destructively adding trees from L to
non-leaf nodes of T with fewer than K branches. Assume that L has enough
items to fill all necessary places. Fill in trees in postorder (bottom to
top, left to right).""

    grafts = iter(L)  # Don't have to use this, but it may be useful.

    def do_grafts(tr):

        if ________________________________:

            for ________________________________:

                ________________________________

            while ____________________ < ____________________:

                ________________________________

                ________________________________

        do_grafts(T)

    return T
```