Functions

Assignment Statements

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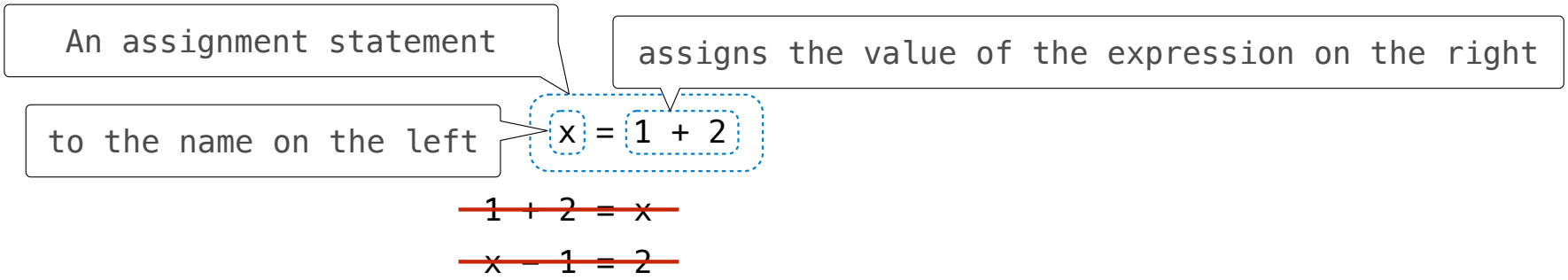
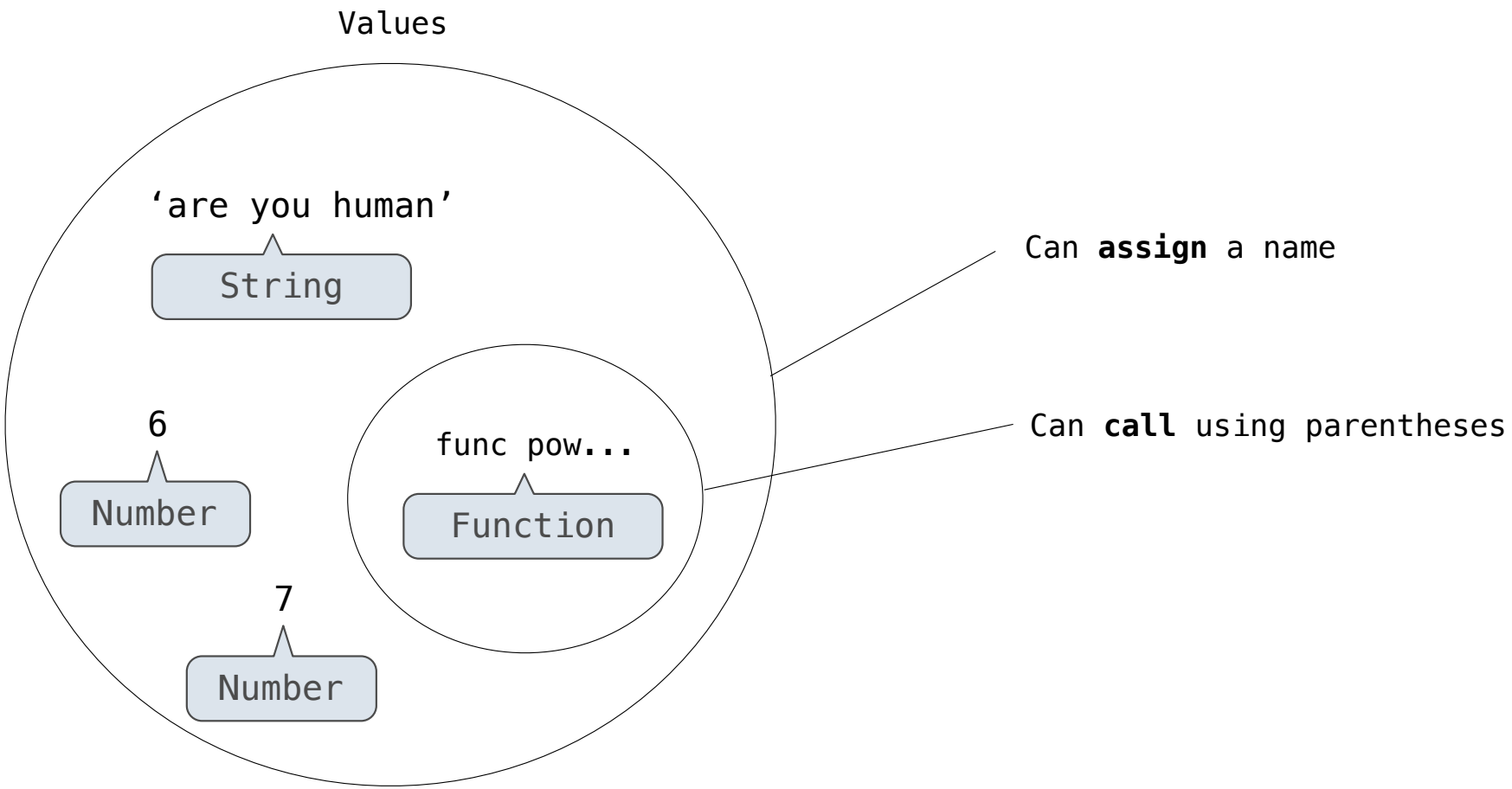


The expression (right) is evaluated, and its value is assigned to the name (left).

```
>>> x = 2
>>> y = x + 1
>>> y
3
>>> x = 5
>>> x
5
>>> y
3
```

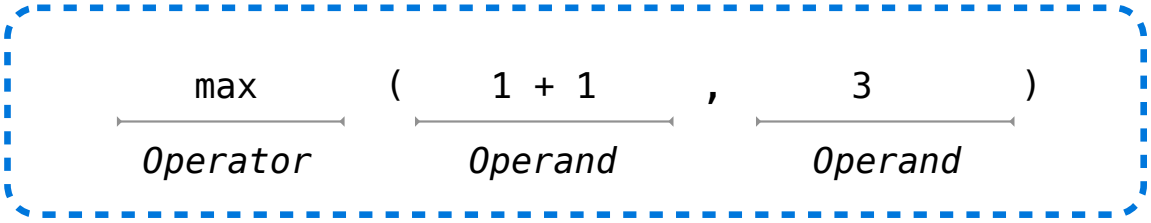
(Demo)

Functions, Values, and Calling



Lecture 1: Anatomy of a Call Expression

Call Expression



Evaluation Procedure for call expressions

(1) Evaluate operator

function

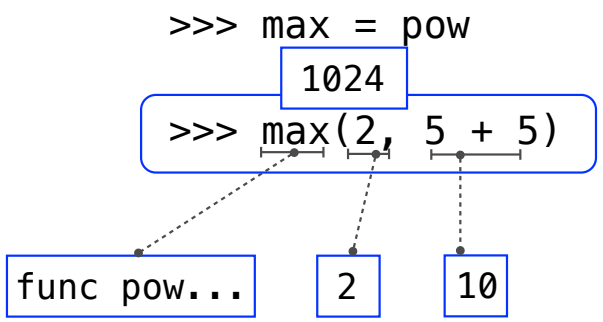
(2) Evaluate each operand

argument

(3) **Apply** the **function** to the **arguments**

Lecture 1: Anatomy of a Call Expression

```
(1)Evaluate operator
(2)Evaluate each operand
(3)Apply the function to
  the arguments
```



(Demo)

Environment Diagrams

Environment Diagrams

(Demo)

Frames

Frame: Holds name-value bindings; looks like a box; no repeated names allowed!

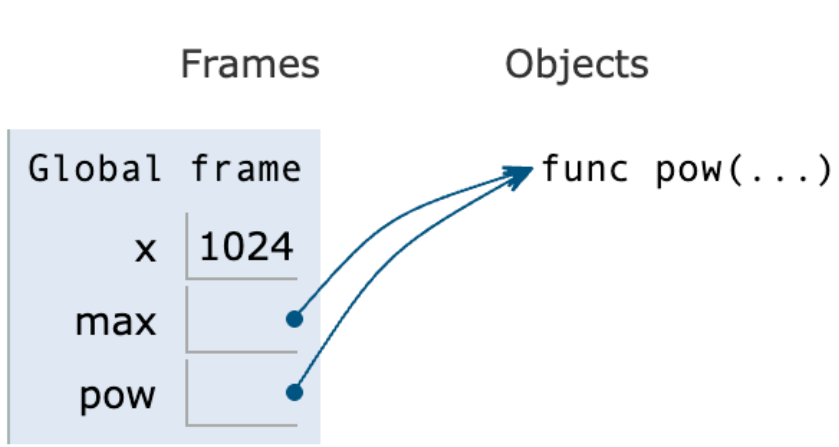
Lookup: Find the value for a name by looking in each frame of an environment

A name (which is a type of expression) such as **x** is evaluated by looking it up

Python 3.6
(known limitations)

```
1  x = pow(2, 10)
2  max = pow
3  x = pow(2, 10)
4  pow = max
5  x = pow(2, 10)
```

Edit this code



User defined functions

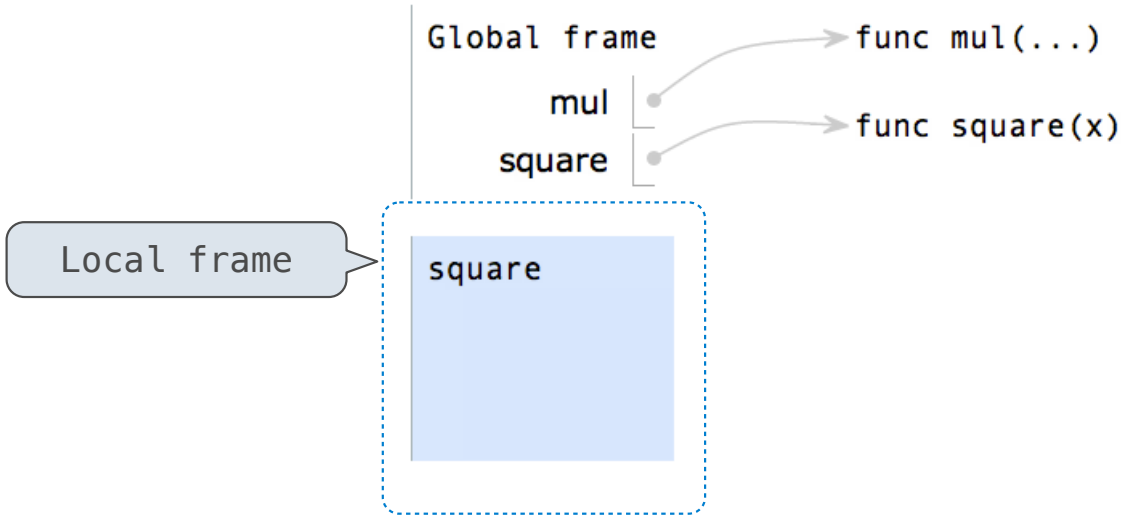
(Demo)

Calling User-Defined Functions

Procedure for calling/applying user-defined functions (version 1):

1. Add a local frame, forming a new environment

```
1  from operator import mul
2  def square(x):
3      return mul(x, x)
4  square(-2)
```



Calling User-Defined Functions

Procedure for calling/applying user-defined functions (version 1):

1. Add a local frame, forming a new environment
2. Bind the function's formal parameters to its arguments in that frame

```
1  from operator import mul
2  def square(x):
3      return mul(x, x)
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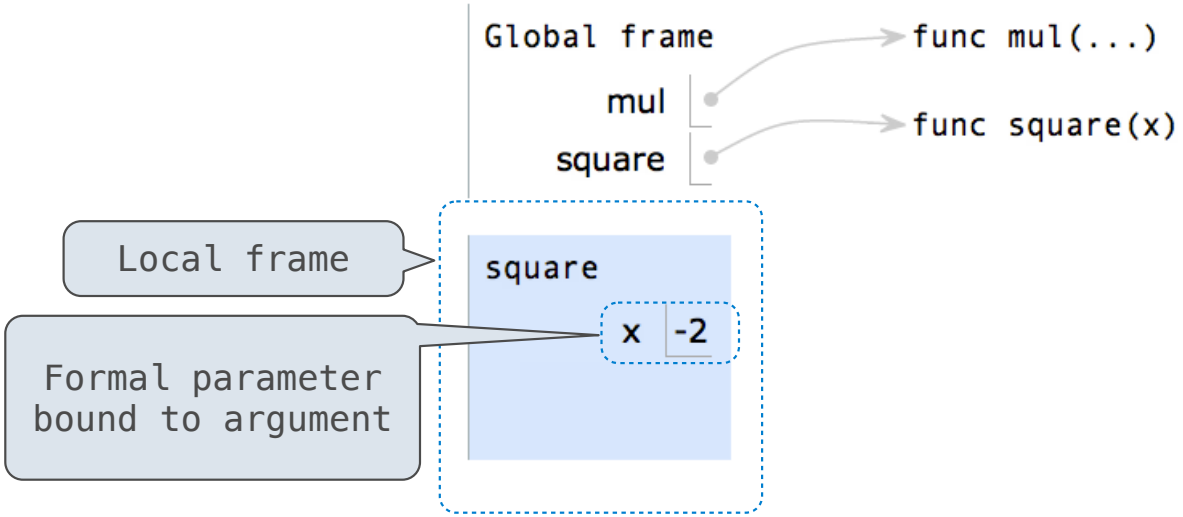

http://pythontutor.com/composingprograms.html#code=from%20operator%20import%20mul%0Adef%20square%28x%29%3A%0A%20%20%20%20return%20mul%28x,%20x%29%0Asquare%28-2%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Calling User-Defined Functions

Procedure for calling/applying user-defined functions (version 1):

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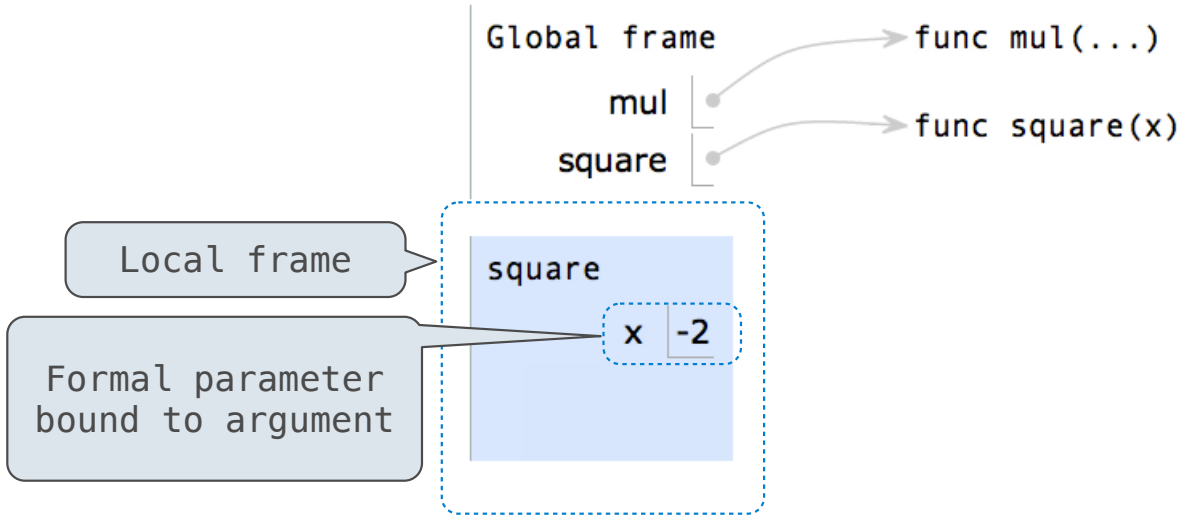

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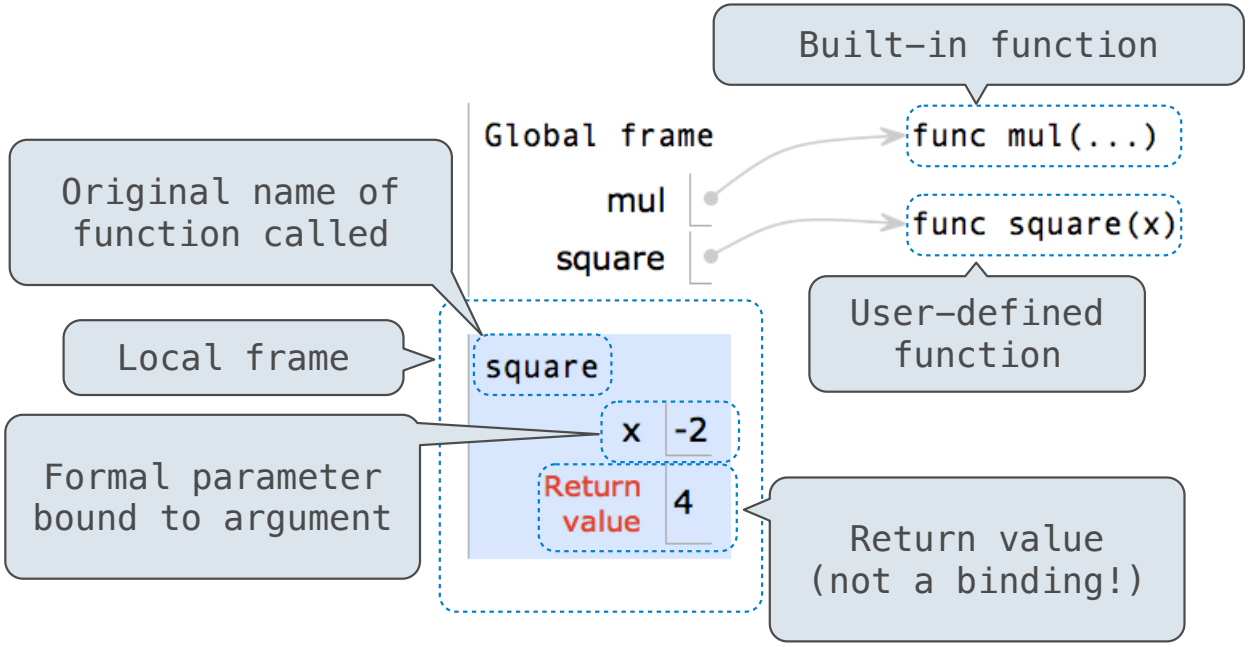


http://pythontutor.com/composingprograms.html#code=from%20operator%20import%20mul%0Adef%20square%28x%29%3A%0A%20%20%20%20return%20mul%28x,%20x%29%0Asquare%28-2%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Frames & Environments

Frame: Holds name-value bindings; looks like a box; no repeated names allowed!

Lookup: Find the value for a name by looking in each frame of an environment

A name (which is an expression) such as **x** is evaluated by looking it up

Global frame: The frame with built-in names (min, pow, etc.)

Environment: A sequence of frames that always ends with the global frame

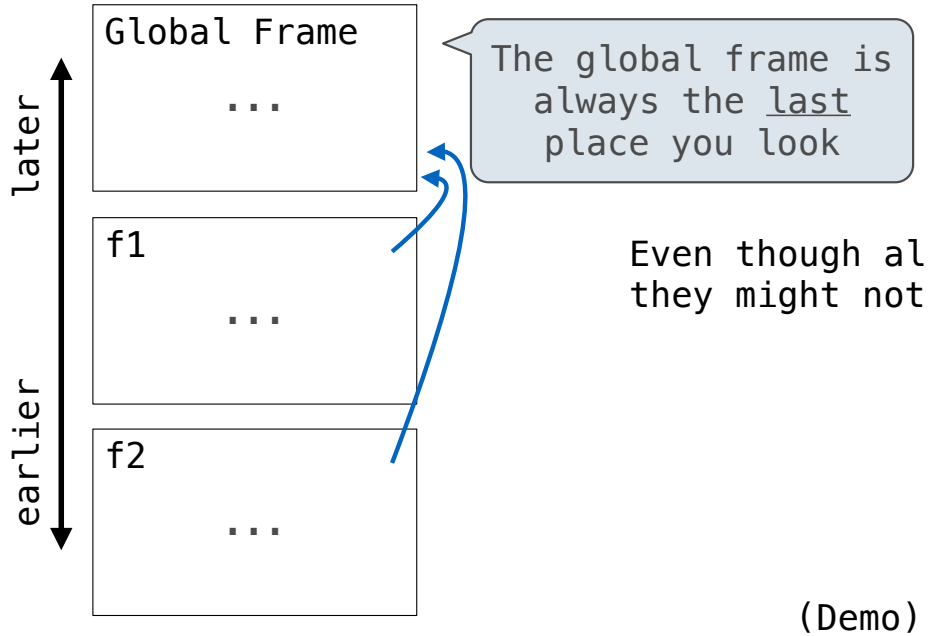
A sequence of frames always has a first frame:

- If the first frame is the global frame, that's the only frame in the environment
- Otherwise, the first frame is local and is followed by the rest of the frames
- Every frame is the first frame of an environment (therefore: one environment per frame)

A Sequence of Frames

An environment is a sequence of frames.

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.



Even though all three frames are in the same **diagram**, they might not be in the same **environment**

(Demo)

Frames & Environments

Why organize information this way?

- Local context before global context
- Calling or returning changes the local context
- Assignment within a function's local frame doesn't affect other frames

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(-2)
```

(Demo)

Print and None

(Demo)

Small Expressions

Problem Definition

From Discussion 0:

Imagine you can call only the following three functions:

- f(x): increment an integer x to get x+1

- g(x): double then decrement an integer x to get 2*x-1

- h(x, y): Concatenates the digits of two different positive integers x and y. For example, h(789, 12) evaluates to 78912 and h(12, 789) evaluates to 12789.

Definition: A *small expression* is a call expression that contains only f, g, h, the number 5, and parentheses. All of these can be repeated. For example, h(f(g(5)), f(f(5))) evaluates to 107.

What's the shortest *small expression* you can find that evaluates to 2026?

Fewest calls?
Shortest length when written?

How do you get to 2026?

5➡9➡10➡19➡20

f(g(f(g(5))))

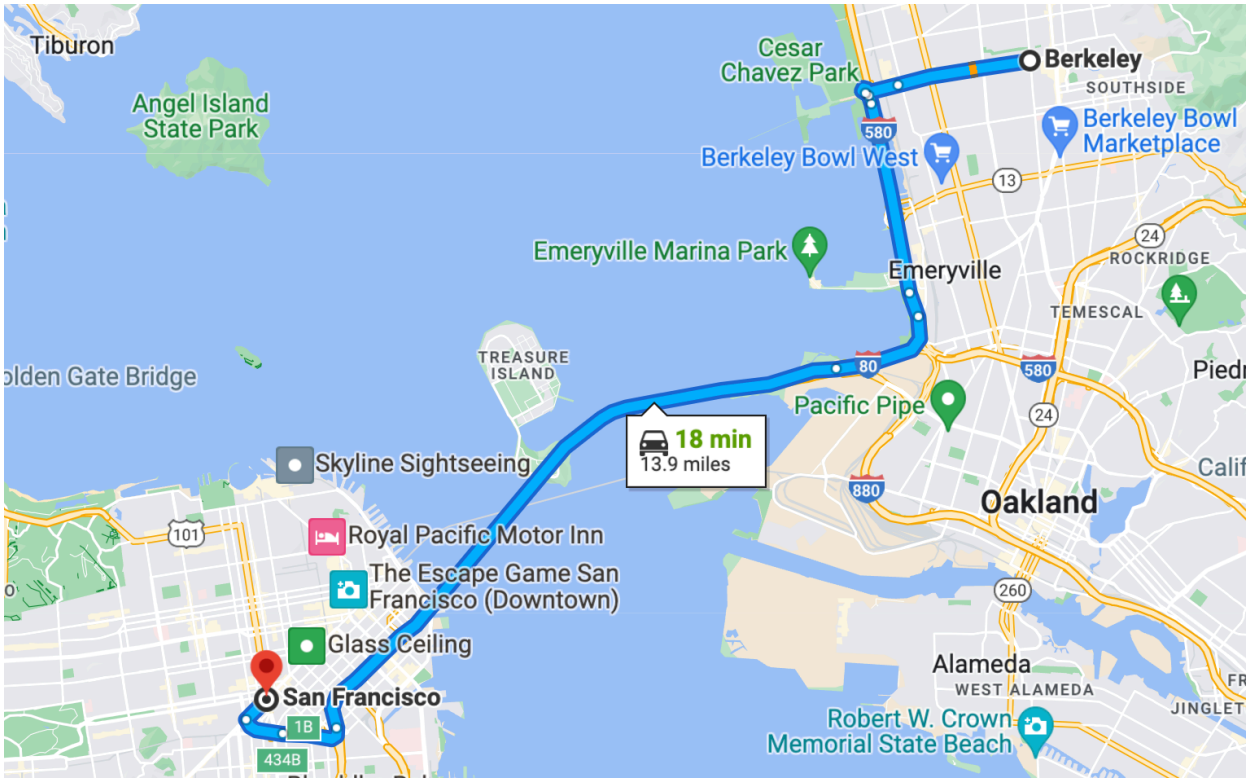
5➡6➡7➡13➡25➡26

f(g(g(f(f(5)))))

h(f(g(f(g(5)))),f(g(g(f(f(5))))))

Effective problem solving:
- Understand the problem
- Come up with ideas
- Turn those ideas into solutions

Search



A common strategy: try a bunch of options to see which is best

Computer programs can evaluate many alternatives by repeating simple operations

A Computational Approach

Try all the small expressions with 3 function calls, then 4 calls, then 5 calls, etc.

```
f(f(f(5))) -> 8
g(f(f(5))) -> 13
f(g(f(5))) -> 12
g(g(f(5))) -> 21
f(f(g(5))) -> 11
g(f(g(5))) -> 19
f(g(g(5))) -> 18
g(g(g(5))) -> 33
```

```
f(f(h(5,5))) -> 57
g(f(h(5,5))) -> 111
f(g(h(5,5))) -> 110
g(g(h(5,5))) -> 217
f(h(5,f(5))) -> 57
g(h(5,f(5))) -> 111
f(h(5,g(5))) -> 60
g(h(5,g(5))) -> 117
f(h(5,h(5,5))) -> 556
g(h(5,h(5,5))) -> 1109
f(h(f(5),5)) -> 66
g(h(f(5),5)) -> 129
f(h(g(5),5)) -> 96
g(h(g(5),5)) -> 189
f(h(h(5,5),5)) -> 556
g(h(h(5,5),5)) -> 1109
```

```
h(5,f(f(5))) -> 57
h(5,g(f(5))) -> 511
h(5,f(g(5))) -> 510
h(5,g(g(5))) -> 517
h(5,f(h(5,5))) -> 556
h(5,g(h(5,5))) -> 5109
h(5,h(5,f(5))) -> 556
h(5,h(5,g(5))) -> 559
h(5,h(5,h(5,5))) -> 5555
h(5,h(f(5),5)) -> 565
h(5,h(g(5),5)) -> 595
h(5,h(h(5,5),5)) -> 5555
h(f(5),f(5)) -> 66
h(f(5),g(5)) -> 69
h(f(5),h(5,5)) -> 655
```

```
h(g(5),f(5)) -> 96
h(g(5),g(5)) -> 99
h(g(5),h(5,5)) -> 955
h(h(5,5),f(5)) -> 556
h(h(5,5),g(5)) -> 559
h(h(5,5),h(5,5)) -> 5555
h(f(f(5)),5) -> 75
h(g(f(5)),5) -> 115
h(f(g(5)),5) -> 105
h(g(g(5)),5) -> 175
h(f(h(5,5)),5) -> 565
h(g(h(5,5)),5) -> 1095
h(h(5,f(5)),5) -> 565
h(h(5,g(5)),5) -> 595
h(h(5,h(5,5)),5) -> 5555
h(h(f(5),5),5) -> 655
h(h(g(5),5),5) -> 955
h(h(h(5,5),5),5) -> 5555
```

Reminder: f(x) increments; g(x) doubles then decrements; h(x, y) concatenates

A Computational Approach

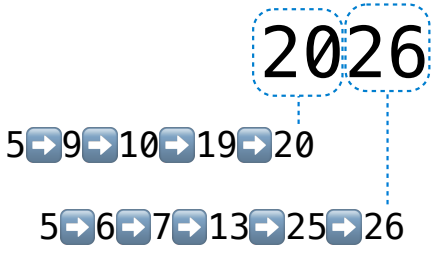
Try all the small expressions with 3 function calls, then 4 calls, then 5 calls, etc.

```
f(g(h(f(g(5)),g(f(f(5))))))) -> 2026 has 8 calls
       10         13
```

```
f(g(f(f(h(f(g(5)),g(f(5))))))) -> 2026 has 9 calls
f(g(f(h(f(g(5)),f(g(f(5))))))) -> 2026 has 9 calls
f(g(h(f(g(5)),f(f(g(f(5))))))) -> 2026 has 9 calls
       10         13
```

```
f(h(f(g(f(f(h(g(5),g(5))))))),5)) -> 2026 has 9 calls
h(f(g(f(f(h(g(5),g(5))))))),f(5)) -> 2026 has 9 calls
            202                   6
```

```
h(f(g(f(g(5)))),f(g(g(f(f(5)))))) -> 2026 has 10 calls
       20          26
```



Reminder: f(x) increments; g(x) doubles then decrements; h(x, y) concatenates

A Computational Approach

Try all the small expressions with 3 function calls, then 4 calls, then 5 calls, etc.

```
def f(x):
    return x + 1
def g(x):
    return 2 * x - 1
def h(x, y):
    return int(str(x) + str(y))

class Number:
    def __init__(self, value):
        self.value = value

    def __str__(self):
        return str(self.value)

    def calls(self):
        return 0

class Call:
    """A call expression."""
    def __init__(self, f, operands):
        self.f = f
        self.operands = operands
        self.value = f(*[e.value for e in operands])

    def __str__(self):
        return f'{self.f.__name__}({",".join(map(str, self.operands))})'

    def calls(self):
        return 1 + sum(o.calls() for o in self.operands)
```

Functions

Containers

Classes

Objects

Sequences

Lazy Evaluation

```
def smalls(n):
    if n == 0:
        yield Number(5)
    else:
        for operand in smalls(n-1):
            yield Call(f, [operand])
            yield Call(g, [operand])
        for k in range(n):
            for first in smalls(k):
                for second in smalls(n-k-1):
                    if first.value > 0 and second.value > 0:
                        yield Call(h, [first, second])

result = []
for i in range(9):
    result.extend([e for e in smalls(i) if e.value == 2025])
```

Generators

Recursion

Tree Recursion

Control

Mutation

Higher-Order Functions

By Midterm 3, you can do this.