

# 1 Recursion and Tree Recursion

## Questions

1.1 What are three things you find in every recursive function?

- 1) Base Case(s)
- 2) Way(s) to reduce the problem into a smaller problem of the same type
- 3) Recursive case(s) that uses the solution of the smaller problem to solve the original (large) problem

1.2 When you write a Recursive function, you seem to call it before it has been fully defined. Why doesn't this break the Python interpreter?

When you define a function, Python does not evaluate the body of the function.

1.3 The **domain** is the type of data that a function takes in as argument. The **range** is the type of data that a function returns. For example, the domain of the function **square** is numbers. The range is numbers.

Below is a Python function that computes the nth Fibonacci number. What's its domain and range? Also identify the three things it contains as a recursive function (from 1.1).

```
def fib(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fib(n-1) + fib(n-2)
```

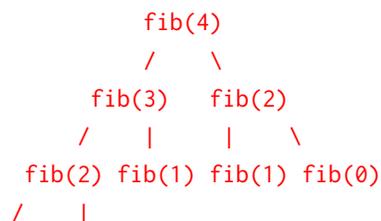
Domain is integers, range is integers.

Base Cases: if n == 0: ..., elif n == 1: ...

Finding Smaller Problems: finding fib(n - 1), fib(n - 2)

Recursive Case: when n is neither 0 nor 1, add together the fib(n - 1) and fib(n - 2) to find fib(n)

1.4 With the definition of the Fibonacci function above, draw out a diagram of the recursive calls made when **fib(4)** is called.



```
fib(1) fib(0)
```

- 1.5 What does the following function **cascade2** do? What is its domain and range?

```
def cascade2(n):  
    print(n)  
    if n >= 10:  
        cascade2(n//10)  
    print(n)
```

Domain is integers, range is None. It takes in a number  $n$  and prints out  $n$ , then prints out  $n$  excluding the ones digit, then prints  $n$  excluding the hundreds digit, and so on, then back up to the full number.

- 1.6 What does each of the the following functions do?

```
def mystery(n):  
    if n == 0:  
        return 0  
    else:  
        return n + mystery(n - 1)
```

sums positive integers up to  $n$  ( $1 + 2 + \dots + n$ )

```
def foo(n):  
    if n <= 1:  
        return n  
    return foo(n - 2) + foo(n - 1)
```

returns the  $n$ th Fibonacci number

```
def fooply(n):  
    if n < 0:  
        return 0  
    return foo(n) + fooply(n - 1)
```

returns the sum of the first  $n$  Fibonacci numbers.

## 2 Higher Order Functions

### Questions

- 2.1 What do lambda expressions do? Can we write all functions as lambda expressions? In what cases are lambda expressions useful?

Lambda expressions create functions. When a lambda expression is evaluated, it produces a function. We often use lambdas to create short anonymous functions that we won't need for too long.

We can't write all functions as lambda expressions because lambda functions all have to have `return` statements and they can't contain very complex multi-line expressions.

- 2.2 Determine if each of the following will error:

```
>>> 1/0
```

Error

```
>>> boom = lambda: 1/0
```

No error, since we don't evaluate the body of the lambda when we define it.

```
>>> boom()
```

Error

- 2.3 Express the following lambda expression using a `def` statement, and the `def` statement using a lambda expression.

```
pow = lambda x, y: x**y
```

```
def pow(x, y):
    return x**y
```

```
def foo(x):
    def f(y):
        def g(z):
            return x + y * z
        return g
    return f
```

```
foo = lambda x: lambda y: lambda z: x + y * z
```

#### 4 *Recursion and Tree Recursion, HOFs*

2.4 Draw Environment Diagrams for the following lines of code

```
square = lambda x: x * x  
higher = lambda f: lambda y: f(f(y))  
higher(square)(5)
```

Solution: <https://goo.gl/LATqV9>

```
a = (lambda f, a: f(a))(lambda b: b * b, 2)
```

Solution: <https://goo.gl/TyriuP>

- 2.5 Write **make\_skipper**, which takes in a number  $n$  and outputs a function. When this function takes in a number  $x$ , it prints out all the numbers between 0 and  $x$ , skipping every  $n$ th number (meaning skip any value that is a multiple of  $n$ ).

```
def make_skipper(n):
    """
    >>> a = make_skipper(2)
    >>> a(5)
    1
    3
    5
    """

    def skipper(x):
        for i in range(x + 1):
            if i % n != 0:
                print(i)
        return skipper
```

- 2.6 Write **make\_alternator** which takes in two functions,  $f$  and  $g$ , and outputs a function. When this function takes in a number  $x$ , it prints out all the numbers between 1 and  $x$ , applying the function  $f$  to every odd-indexed number and  $g$  to every even-indexed number before printing.

```
def make_alternator(f, g):
    """
    >>> a = make_alternator(lambda x: x * x, lambda x: x + 4)
    >>> a(5)
    1
    6
    9
    8
    25
    >>> b = make_alternator(lambda x: x * 2, lambda x: x + 2)
    >>> b(4)
    2
    4
    6
    6
    """

    def alternator(n):
        i = 1
        while i <= n:
            if i % 2 == 1:
                print(f(i))
            else:
                print(g(i))
            i += 1
```

```
    i += 1  
    return alternator
```