1 Sequences

Questions

1.1 What would Python display?

```python
lst = [1, 2, 3, 4, 5]
lst[1:3]

lst[0:len(lst)]

lst[-4:]

lst[:3]

lst[3:]

lst[:]

lst[1:4:2]

lst[0:4:3]

lst[:4:2]

lst[1::2]

lst[::2]

lst[::-1]

lst2 = [6, 1, 0, 7]
lst + lst2

lst + 100

lst3 = [[1], [2], [3]]
lst + lst3
```
1.2 Draw the environment diagram that results from running the code below

```python
def reverse(lst):
    if len(lst) <= 1:
        return lst
    return reverse(lst[1:]) + [lst[0]]

lst = [1, [2, 3], 4]
rev = reverse(lst)
```

```
2 Mutability

Questions

2.1 Name two data types that are mutable. What does it mean to be mutable?

2.2 Name at least two data types that are not mutable.

2.3 Will the following code error? If so, why?

```python
a = 1
b = 2
dt = {a: 1, b: 2}

a = [1]
b = [2]
dt = {a: 1, b: 2}
```

2.4 Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write Error, but include all output displayed before the error.

```python
a = [1, [2, 3], 4]
c = a[1]
c

a.append(c)
a

c[0] = 0
c

a

a.extend(c)
c[1] = 9
a

list1 = [1, 2, 3]
list2 = [1, 2, 3]
list1 == list2

list1 is list2
```
3 Data Abstraction

Questions

3.1 Why are Abstract Data Types useful?

3.2 What are the two types of functions necessary to make an Abstract Data Type? What do they do?

3.3 What is a Data Abstraction Violation? Why is it a terrible sin to commit?

3.4 Assume that \texttt{rational}, \texttt{numer}, \texttt{denom}, and \texttt{gcd} run without error and behave as described below. Can you identify where the abstraction barrier is broken? Come up with a scenario where this code runs without error and a scenario where this code would stop working.

```python
def rational(num, den): # Returns a rational number ADT
    #implementation not shown

def numer(x): # Returns the numerator of the given rational
    #implementation not shown

def denom(x): # Returns the denominator of the given rational
    #implementation not shown

def gcd(a, b): # Returns the GCD of two numbers
    #implementation not shown

def simplify(f1): #Simplifies a rational number
    g = gcd(f1[0], f1[1])
    return rational(numer(f1) // g, denom(f1) // g)

def multiply(f1, f2): # Multiples and simplifies two rational numbers
    r = rational(numer(f1) * numer(f2), denom(f1) * denom(f2))
    return simplify(r)

x = rational(1, 2)
y = rational(2, 3)
multiply(x, y)
```
4 Trees

Questions

4.1 Fill in this implementation of the Tree ADT.

```python
def tree(label, branches = []):
    for b in branches:
        assert is_tree(b), 'branches must be trees'
    return [label] + list(branches)

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for b in branches(tree):
        if not is_tree(b):
            return False
    return True

def label(tree):

def branches(tree):

def is_leaf(tree):
```

4.2 A min-heap is a tree with the special property that every node's value is less than or equal to the values of all of its children. For example, the following tree is a min-heap:

```
1
/ | \  
5 3 6
| / \  
7 9 4
```

However, the following tree is not a min-heap because the node with value 3 has a value greater than one of its children:

```
1
/ | \  
5 3 6
| / \  
7 9 2
```
Write a function `is_min_heap` that takes a tree and returns True if the tree is a min-heap and False otherwise.

```python
def is_min_heap(t):
```

4.3 Write a function `largest_product_path` that finds the largest product path possible. A product path is defined as the product of all nodes between the root and a leaf. The function takes a tree as its parameter. Assume all nodes have a non-negative value.

```
    3
   / | \
  7 8 4
 / | |
2 1
```

For example, calling `largest_product_path` on the above tree would return 42, since $3 \times 7 \times 2$ is the largest product path.

```python
def largest_product_path(tree):
    
    >>> largest_product_path(None)
    0
    >>> largest_product_path(tree(3))
    3
    >>> t = tree(3, [tree(7, [tree(2)]), tree(8, [tree(1)]), tree(4)])
    >>> largest_product_path(t)
    42
    
```
Challenge Question: The level-order traversal of a tree is defined as visiting the nodes in each level of a tree before moving onto the nodes in the next level. For example, the level order of the following tree is: 3 7 8 4

```
  3
 / | \ 
7  8  4
```

Write a function `level_order` that takes in a tree as the parameter and returns a list of the values of the nodes in level order.

```python
def level_order(tree):
```