

**INSTRUCTIONS**

- You have 10 minutes to complete this quiz.
- The exam is closed book, closed notes, closed computer, closed calculator.
- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.
- For multiple choice questions, fill in each option or choice completely.
  - means mark **all options** that apply
  - means mark a **single choice**

Last name	
First name	
Student ID number	
CalCentral email (_@berkeley.edu)	
Discussion Section	_____
<i>All the work on this exam is my own.</i> <b>(please sign)</b>	

0. **Your thoughts?** What can we do to make your learning experience better?

## 1. Copy Machine

- (a) Peter wants to print this week's discussion handouts for all the students in CS 61A. However, both printers are broken! The first printer only prints multiples of  $n$  pages, and the second printer only prints multiples of  $m$  pages. Help Peter figure out whether or not it's possible to print exactly  $total$  number of handouts!

```
def has_sum(total, n, m):
    """
    >>> has_sum(1, 3, 5)
    False
    >>> has_sum(5, 3, 5) # 0 * 3 + 1 * 5 = 5
    True
    >>> has_sum(11, 3, 5) # 2 * 3 + 1 * 5 = 11
    True
    """

    if total == 0: # (total == n or total == m) works too except when total equals 0

        return True

    elif total < 0: # (total < min(n1, n2)) works given alternate base case

        return False

    return has_sum(total - n, n, m) or has_sum(total - m, n, m)
```

- (b) The next day, the printers break down even more! Each time they are used, the first printer prints a random  $x$  copies  $50 \leq x \leq 60$ , and the second printer prints a random  $y$  copies  $130 \leq y \leq 140$ . Peter also relaxes his expectations: he's satisfied as long as there's at least  $lower$  copies so there are enough for everyone, but no more than  $upper$  copies to prevent waste.

```
def sum_range(lower, upper):
    """
    >>> sum_range(45, 60) # Printer 1 prints within this range
    True
    >>> sum_range(40, 55) # Printer 1 can print a number 56-60
    False
    >>> sum_range(170, 201) # Printer 1 + 2 will print between 180 and 200 copies total
    True
    """

    def copies(pmin, pmax):

        if lower <= pmin and pmax <= upper:

            return True

        elif upper < pmin:

            return False

        return copies(pmin + 50, pmax + 60) or copies(pmin + 130, pmax + 140)

    return copies(0, 0)
```